Zadatak 9. Izračunaj:

1) $\left[\sqrt{3}\left(\cos\frac{\pi}{12}-i\sin\frac{5\pi}{12}\right)\right]^{-6}$;

2) $\left(-\frac{1}{2}\cos\frac{2\pi}{15}-\frac{i}{2}\sin\frac{4\pi}{15}\right)^{-5}$.

Rješenje. **1)**

$$\begin{aligned}
\left[\sqrt{3}\left(\cos\frac{\pi}{12}-i\sin\frac{5\pi}{12}\right)\right]^{-6} &= \left[\sqrt{3}\left(\cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)-i\sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)\right)\right]^{-6}\\
&= \left[\sqrt{3}\left(\left(\cos\frac{\pi}{4}\cos\frac{\pi}{6}+\sin\frac{\pi}{4}\sin\frac{\pi}{6}\right)-\left(\sin\frac{\pi}{4}\cos\frac{\pi}{6}+\cos\frac{\pi}{4}\sin\frac{\pi}{6}\right)i\right)\right]^{-6}\\
&= \left[\sqrt{3}\left(\left(\cos\frac{\pi}{4}\cos\frac{\pi}{6}+\sin\frac{\pi}{4}\sin\frac{\pi}{6}\right)-\left(\cos\frac{\pi}{4}\cos\frac{\pi}{6}+\sin\frac{\pi}{4}\sin\frac{\pi}{6}\right)i\right)\right]^{-6}\\
&= \left[\sqrt{3}\left(\cos\frac{\pi}{4}\cos\frac{\pi}{6}+\sin\frac{\pi}{4}\sin\frac{\pi}{6}\right)(1-i)\right]^{-6}\\
&= \frac{1}{27\cos^6\frac{\pi}{12}}\left[\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)\right]^{-6}\\
&= \frac{1}{27\cos^6\frac{\pi}{12}}\cdot\frac{1}{8}\left(\cos\left(-\frac{\pi}{2}\right)+i\sin\left(-\frac{\pi}{2}\right)\right)\\
&= \frac{1}{6^3\cos^6\frac{\pi}{12}}\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right) = -\frac{i}{6^3\cos^6\frac{\pi}{12}}.
\end{aligned}$$