Zadatak 10. Zapiši u trigonometrijskom obliku broj:

1) $\left(\frac{2}{i\sqrt{3}-1}\right)^6$;

2) $\frac{i-1}{\sqrt{3}+i}$;

3) $\frac{1}{1+i}+\frac{1}{1-i}$;

4) $\frac{1}{1-i\sqrt{3}}-\frac{1}{i\sqrt{3}+1}$.

Rješenje. **1)** $\left(\frac{2}{i\sqrt{3}-1}\right)^6=\left(\frac{2}{-1+i\sqrt{3}}\cdot\frac{-1-i\sqrt{3}}{-1-i\sqrt{3}}\right)^6=\left[\frac{-2(1+i\sqrt{3})}{1+3}\right]^6=\left[-\frac{1}{2}(1+i\sqrt{3})\right]^6=\frac{1}{2^6}\left[2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]^6=\cos 2\pi+i\sin 2\pi=\cos 0+i\sin 0=1$;

2) $\frac{i-1}{\sqrt{3}+i}=\frac{-1+i}{\sqrt{3}+i}=\frac{\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)}{2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)}=\frac{\sqrt{2}}{2}\left(\cos\left(\frac{3\pi}{4}-\frac{\pi}{6}\right)+i\sin\left(\frac{3\pi}{4}-\frac{\pi}{6}\right)\right)=\frac{\sqrt{2}}{2}\left(\cos\frac{7\pi}{12}+i\sin\frac{7\pi}{12}\right)$;

3) $\frac{1}{1+i}+\frac{1}{1-i}=\frac{1-i+1+i}{1-i^2}=\frac{2}{2}=1=\cos 0+i\sin 0$;

4) $\frac{1}{1-i\sqrt{3}}-\frac{1}{i\sqrt{3}+1}=\frac{1+i\sqrt{3}-1+i\sqrt{3}}{(1-i\sqrt{3})(1+i\sqrt{3})}=\frac{2i\sqrt{3}}{1+3}=\frac{\sqrt{3}}{2}i=\frac{\sqrt{3}}{2}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$.