Zadatak 11. Odredi argument i modul kompleksnog broja:

1) $z=\dfrac{1+i\sqrt{3}}{2i\left(\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}\right)}$;

2) $z=\dfrac{i-1}{\cos\dfrac{5\pi}{6}+i\sin\dfrac{5\pi}{6}}$;

3) $z=(\sqrt{2}-i)^{10}\cdot(1-i\sqrt{2})^{20}$;

4) $z=\dfrac{(1-i\sqrt{3})^{15}}{-\sqrt{2}\left(\cos\dfrac{\pi}{6}-i\sin\dfrac{17\pi}{6}\right)}$.

Rješenje. **1)**

$$\begin{aligned}
z&=\frac{1+i\sqrt{3}}{2i\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}\cdot\frac{-i}{-i}=\frac{\frac{\sqrt{3}}{2}-\frac{1}{2}i}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}\\
&=\frac{\cos\frac{11\pi}{6}+i\sin\frac{11\pi}{6}}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}=\cos\left(\frac{11\pi}{6}-\frac{\pi}{3}\right)+i\sin\left(\frac{11\pi}{6}-\frac{\pi}{3}\right)\\
&=\cos\frac{9\pi}{6}+i\sin\frac{9\pi}{6}=\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\\
&\Longrightarrow\ |z|=1,\ \arg z=\frac{3\pi}{2}.
\end{aligned}$$

2)

$$\begin{aligned}
z&=\frac{i-1}{\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}}=\frac{\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}}{\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}}\\
&=\cos\left(\frac{3\pi}{4}-\frac{5\pi}{6}\right)+i\sin\left(\frac{3\pi}{4}-\frac{5\pi}{6}\right)\\
&=\cos\left(-\frac{\pi}{12}\right)+i\sin\left(-\frac{\pi}{12}\right)=\cos\frac{23\pi}{24}+i\sin\frac{23\pi}{24}\\
&\Longrightarrow\ |z|=1,\ \arg z=\frac{23\pi}{24}.
\end{aligned}$$

3)

$$\begin{aligned}
z &= (\sqrt{2}-i)^{10}(1-i\sqrt{2})^{20} = (\sqrt{2}-i)^{10}\left[\frac{(1-i\sqrt{2})i}{i}\right]^{20}\\
&= (\sqrt{2}-i)^{10}\left(\frac{i+\sqrt{2}}{i}\right)^{20} = (\sqrt{2}-i)^{10}(\sqrt{2}+i)^{20}\\
&= \left[\sqrt{3}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)\right]^{10}\left[\sqrt{3}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]^{20}\\
&= 3^5\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)3^{10}(\cos\pi+i\sin\pi)\\
&= 3^{15}\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)\\
&\implies |z| = 3^{15},\ \arg z = \frac{3\pi}{2}.
\end{aligned}$$

4)

$$\begin{aligned}
z &= \frac{(1-i\sqrt{3})^{15}}{\sqrt{2}\left(\cos\frac{\pi}{6}-i\sin\frac{17\pi}{6}\right)} = \frac{\left[2\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)\right]^{15}}{-\sqrt{2}\left(\cos\frac{11\pi}{6}+i\sin\frac{11\pi}{6}\right)}\\
&= \frac{2^{15}(\cos\pi+i\sin\pi)}{-\sqrt{2}\left(\cos\frac{11\pi}{6}+i\sin\frac{11\pi}{6}\right)} = -2^{14}\sqrt{2}\left(\cos\left(\pi-\frac{11\pi}{6}\right)+i\sin\left(\pi-\frac{11\pi}{6}\right)\right)\\
&= -2^{14}\sqrt{2}\left(\cos\left(-\frac{5\pi}{6}\right)+i\sin\left(-\frac{5\pi}{6}\right)\right) = -2^{\frac{29}{2}}\left(\cos\frac{7\pi}{6}+i\sin\frac{7\pi}{6}\right)\\
&= 2^{\frac{29}{2}}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\\
&\implies |z| = 2^{\frac{29}{2}},\ \arg z = \frac{\pi}{6}.
\end{aligned}$$