Zadatak 12. Koristeći de Moivreovu formulu dokaži sljedeće identitete:

1) $\cos 3\alpha = \cos^3\alpha - 3\cos\alpha\sin^2\alpha\,;$

2) $\sin 3\alpha = 3\cos^2\alpha\sin\alpha - \sin^3\alpha\,;$

3) $\cos 4\alpha = \cos^4\alpha - 6\cos^2\alpha\sin^2\alpha + \sin^4\alpha\,;$

4) $\sin 4\alpha = 4\cos^3\alpha\sin\alpha - 4\cos\alpha\sin^3\alpha\,.$

Rješenje. **1)** i **2)**

$$\begin{aligned}\cos 3\alpha + i\sin 3\alpha &= (\cos\alpha + i\sin\alpha)^3\\ &= \cos^3\alpha + 3\cos^2\alpha\sin\alpha\cdot i - 3\cos\alpha\sin^2\alpha - i\sin^3\alpha\\ &= [\cos^3\alpha - 3\cos\alpha\sin^2\alpha] + [3\cos^2\alpha\sin\alpha - \sin^3\alpha]i\end{aligned}$$

$$\Longrightarrow \begin{aligned}\cos 3\alpha &= \cos^3\alpha - 3\cos\alpha\sin^2\alpha\\ \sin 3\alpha &= 3\cos^2\alpha\sin\alpha - \sin^3\alpha\end{aligned}$$

3) i **4)**

$$\begin{aligned}\cos 4\alpha + i\sin 4\alpha &= (\cos\alpha + i\sin\alpha)^4\\ &= \cos^4\alpha + 4\cos^3\alpha\sin\alpha\cdot i - 6\cos^2\alpha\sin^2\alpha - 4\cos\alpha\sin^3\alpha\cdot i + \sin^4\alpha\\ &= [\cos^4\alpha - 6\cos^2\alpha\sin^2\alpha + \sin^4\alpha] + [4\cos^3\alpha\sin\alpha - 4\cos\alpha\sin^3\alpha]i\end{aligned}$$

$$\Longrightarrow \begin{aligned}\cos 4\alpha &= \cos^4\alpha - 6\cos^2\alpha\sin^2\alpha + \sin^4\alpha\\ \sin 4\alpha &= 4\cos^3\alpha\sin\alpha - 4\cos\alpha\sin^3\alpha\end{aligned}$$