Zadatak 13. Ako je $x + \dfrac{1}{x} = 2\cos\alpha$, dokaži da je $x^n + \dfrac{1}{x^n} = 2\cos n\alpha$.

Rješenje.

$$
\begin{aligned}
x + \frac{1}{x} &= 2\cos\alpha \implies x^2 - 2x\cos\alpha + 1 = 0\\
x_{1,2} &= \frac{2\cos\alpha \pm \sqrt{4\cos^2\alpha - 4}}{2} = \frac{2\cos\alpha \pm \sqrt{4(\cos^2\alpha - 1)}}{2}\\
&= \frac{2[\cos\alpha \pm i\sin\alpha]}{2} = \cos\alpha \pm i\sin\alpha\\
x^n &= (\cos\alpha \pm i\sin\alpha)^n = \cos n\alpha \pm i\sin n\alpha\\
\frac{1}{x^n} &= x^{-n} = (\cos\alpha \pm i\sin\alpha)^{-n} = \cos(-n\alpha) \pm i\sin(-n\alpha)\\
&= \cos n\alpha \mp i\sin n\alpha\\
x^n + \frac{1}{x^n} &= \cos n\alpha \pm i\sin n\alpha + \cos n\alpha \mp i\sin\alpha = 2\cos n\alpha.
\end{aligned}
$$