Zadatak 14. Izračunaj naznačene korijene:

1) $\sqrt{-i}$; **2)** $\sqrt{3+4i}$; **3)** $\sqrt{1+i\sqrt{3}}$.

Rješenje.

$$\sqrt[n]{z} = \sqrt[n]{|z|}\left(\cos\frac{\varphi+2k\pi}{n} + i\sin\frac{\varphi+2k\pi}{n}\right),\quad n\in\mathbf{Z}_n$$

1) $\sqrt{-i}$,

$$a=0,\ b=-1,\ |z|=1,\ \operatorname{tg}\varphi=-1 \implies \varphi=\frac{3\pi}{2}$$

$$w_k=\sqrt{-i}=\sqrt{1}\left(\cos\frac{\frac{3\pi}{2}+2k\pi}{2}+i\sin\frac{\frac{3\pi}{2}+2k\pi}{2}\right),\ k\in\mathbf{Z}_2$$

$$w_0=\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}=-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$$

$$w_1=\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i.$$

2) $\sqrt{3+4i}$,

$$\sqrt{3+4i}=\sqrt{4+4i-1}=\sqrt{2^2+2\cdot 2i+i^2}=\sqrt{[\pm(2+i)]^2}=\pm(2+i)$$

3) $\sqrt{1+i\sqrt{3}}$,

$$a=1,\ b=\sqrt{3},\ |z|=2,\ \operatorname{tg}\varphi=\sqrt{3} \implies \varphi=\frac{\pi}{3}$$

$$w_k=\sqrt{1+i\sqrt{3}}=\sqrt{2}\left(\cos\frac{\frac{\pi}{3}+2k\pi}{2}+i\sin\frac{\frac{\pi}{3}+2k\pi}{2}\right),\ k\in\mathbf{Z}_2$$

$$=\sqrt{2}\left(\cos\left(\frac{\pi}{6}+k\pi\right)+i\sin\left(\frac{\pi}{6}+k\pi\right)\right),\ k\in\mathbf{Z}_2$$

$$w_0=\sqrt{2}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)$$

$$w_1=\sqrt{2}\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}i\right).$$