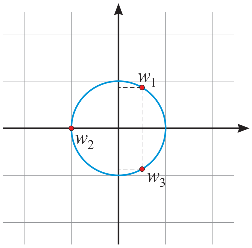
**Zadatak 15.** Nađi sve vrijednosti korijena i predoči ih u Gaussovoj ravnini:

**1)** $\sqrt[3]{-1}$; **2)** $\sqrt[3]{8}$; **3)** $\sqrt[3]{-27}$;
**4)** $\sqrt[4]{i}$; **5)** $\sqrt[6]{1}$. **6)** $\sqrt[3]{i}$;
**7)** $\sqrt[3]{-2+2i}$; **8)** $\sqrt[4]{-1}$; **9)** $\sqrt[6]{-1}$;
**10)** $\sqrt[8]{-1}$.

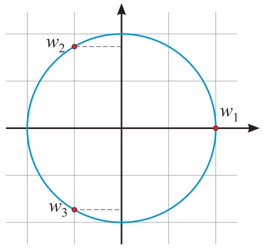
***Rješenje.*** **1)** $\sqrt[3]{-1}$,

$$a=-1,\ b=0,\ |z|=1,\ \operatorname{tg}\varphi=0 \implies \varphi=\pi$$
$$w=\sqrt[3]{-1}=\sqrt[3]{1}\Big(\cos\frac{\pi+2k\pi}{3}+i\sin\frac{\pi+2k\pi}{3}\Big),\ k\in\mathbf{Z}_3$$
$$w=\cos\frac{1+2k}{3}\pi+i\sin\frac{1+2k}{3}\pi,\ k\in\mathbf{Z}_3$$
$$w_1=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}=\frac{1}{2}+\frac{\sqrt{3}}{2}i$$
$$w_2=\cos\pi+i\sin\pi=-1$$
$$w_3=\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}=\frac{1}{2}-\frac{\sqrt{3}}{2}i$$



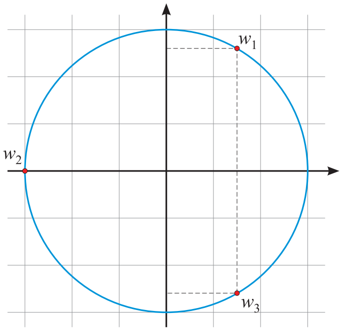

**2)** $\sqrt[3]{8}$,

$$a=8,\ b=0,\ |z|=8,\ \operatorname{tg}\varphi=0 \implies \varphi=0$$
$$w=\sqrt[3]{8}=2\Big(\cos\frac{0+2k\pi}{3}+i\sin\frac{0+2k\pi}{3}\Big),\ k\in\mathbf{Z}_3$$
$$w=2\Big(\cos\frac{2k\pi}{3}+i\sin\frac{2k\pi}{3}\Big),\ k\in\mathbf{Z}_3$$
$$w_1=2(\cos 0+i\sin 0)=2$$
$$w_2=2\Big(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\Big)=-1+i\sqrt{3}$$
$$w_3=2\Big(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\Big)=-1-i\sqrt{3}$$



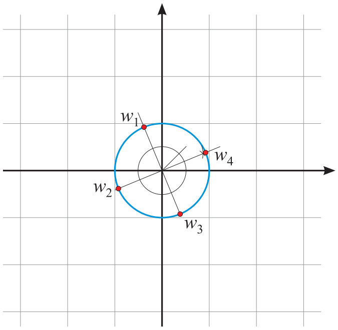

**3)** $\sqrt[3]{-27}$,

$$a=-27,\ b=0,\ |z|=27,\ \operatorname{tg}\varphi=0 \implies \varphi=\pi$$
$$w=\sqrt[3]{-27}=\sqrt[3]{27}\Big(\cos\frac{\pi+2k\pi}{3}+i\sin\frac{\pi+2k\pi}{3}\Big),\ k\in\mathbf{Z}_3$$
$$w=3\Big(\cos\frac{1+2k}{3}\pi+i\sin\frac{1+2k}{3}\pi\Big),\ k\in\mathbf{Z}_3$$
$$w_1=3\Big(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\Big)=\frac{3}{2}+\frac{3\sqrt{3}}{2}i$$
$$w_2=3(\cos\pi+i\sin\pi)=-3$$
$$w_3=3\Big(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\Big)=\frac{3}{2}-\frac{3\sqrt{3}}{2}i$$




**4)** $\sqrt[4]{i}$,

$$a=0,\ b=1,\ |z|=1,\ \operatorname{tg}\varphi=\infty \implies \varphi=\frac{\pi}{2}$$
$$w=\sqrt[4]{i}=\sqrt[4]{1}\left(\cos\frac{\frac{\pi}{2}+2k\pi}{4}+i\sin\frac{\frac{\pi}{2}+2k\pi}{4}\right),\ k\in\mathbf{Z}_4$$
$$w=\cos\Big(\frac{\pi}{8}+k\frac{\pi}{2}\Big)+i\sin\Big(\frac{\pi}{8}+k\frac{\pi}{2}\Big),\ k\in\mathbf{Z}_4$$




**5)** $\sqrt[6]{1}$,

$$a=1,\ b=0,\ |z|=1,\ \varphi=0$$

$$w=\sqrt[6]{1}=\cos\frac{2k\pi}{6}+i\sin\frac{2k\pi}{6},\ k\in\mathbf{Z}_6$$

$$w=\cos\frac{k\pi}{3}+i\sin\frac{k\pi}{3},\ k\in\mathbf{Z}_6$$

$$w_1=\cos 0+i\sin 0=1$$

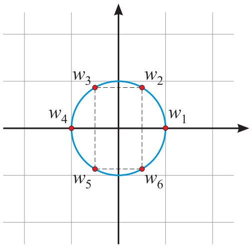
$$w_2=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}=\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

$$w_3=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}=-\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

$$w_4=\cos\pi+i\sin\pi=-1$$

$$w_5=\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$$

$$w_6=\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}=\frac{1}{2}-\frac{\sqrt{3}}{2}i$$



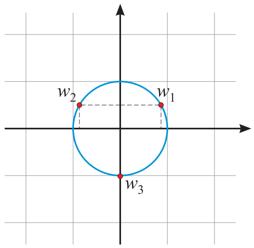

**6)** $\sqrt[3]{i}$,

$$a=0,\ b=1,\ |z|=1,\ \varphi=\frac{\pi}{2}$$

$$w=\cos\frac{\frac{\pi}{2}+2k\pi}{3}+i\sin\frac{\frac{\pi}{2}+2k\pi}{3},\ k\in\mathbf{Z}_3$$

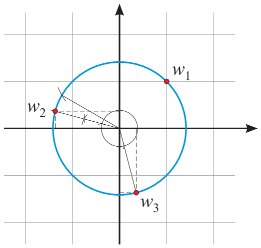
$$w=\cos\left(\frac{\pi}{6}+\frac{4k\pi}{6}\right)+i\sin\left(\frac{\pi}{6}+\frac{4k\pi}{6}\right),\ k\in\mathbf{Z}_3$$

$$w_1 = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{1}{2}i$$
$$w_2 = \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$$
$$w_3 = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = -i$$




**7)** $\sqrt[3]{-2+2i}$,

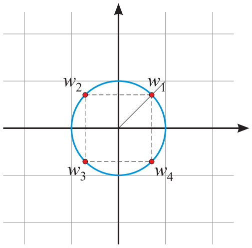
$$a=-2,\ b=2,\ |z|=\sqrt{8},\ \operatorname{tg}\varphi=-1 \implies \varphi=\frac{3\pi}{4}$$
$$w=\sqrt[3]{-2+2i}=\sqrt[3]{\sqrt{8}}\left(\cos\frac{\frac{3\pi}{4}+2k\pi}{3}+i\sin\frac{\frac{3\pi}{4}+2k\pi}{3}\right),\ k\in\mathbf{Z}_3$$
$$w=\sqrt{2}\left(\cos\frac{3+8k}{12}\pi+i\sin\frac{3+8k}{12}\pi\right),\ k\in\mathbf{Z}_3$$
$$w_1=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)=1+i$$
$$w_2=\sqrt{2}\left(\cos\frac{11\pi}{12}+i\sin\frac{11\pi}{12}\right)$$
$$w_3=\sqrt{2}\left(\cos\frac{19\pi}{12}+i\sin\frac{19\pi}{12}\right)$$




**8)** $\sqrt[4]{-1}$,

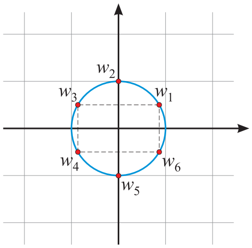
$$a=-1,\ b=0,\ |z|=1,\ \varphi=\pi$$
$$w=\sqrt[4]{-1}=\cos\frac{\pi+2k\pi}{4}+i\sin\frac{\pi+2k\pi}{4},\ k\in\mathbf{Z}_4$$
$$w_1=\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$$

$$w_2 = \cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$$
$$w_3 = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$$
$$w_4 = \cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$$



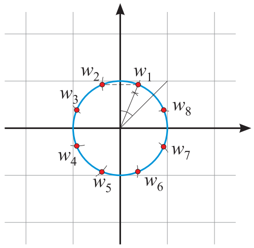

**9)** $\sqrt[6]{-1}$,

$$w = \sqrt[6]{-1} = \cos\frac{\pi + 2k\pi}{6} + i\sin\frac{\pi + 2k\pi}{6},\ k \in \mathbf{Z}_6$$
$$w_1 = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + \frac{1}{2}i$$
$$w_2 = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$$
$$w_3 = \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$$
$$w_4 = \cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6} = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$$
$$w_5 = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = -i$$
$$w_6 = \cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6} = \frac{\sqrt{3}}{2} - \frac{1}{2}i$$




**10)** $\sqrt[8]{-1}$,

$$w = \sqrt[8]{-1} = \cos\frac{\pi + 2k\pi}{8} + i\sin\frac{\pi + 2k\pi}{8},\ k \in \mathbf{Z}_8$$



$w_1$
$w_2$
$w_3$
$w_4$
$w_5$
$w_6$
$w_7$
$w_8$