Zadatak 16. Odredi sve vrijednosti sljedećih korijena:

1) $\sqrt[4]{-\frac{1}{2}-i\frac{\sqrt{3}}{2}}$; **2)** $\sqrt[3]{2+2i}$; **3)** $\sqrt[6]{\frac{1-i}{\sqrt{3}+i}}$.

Rješenje. **1)** $\sqrt[4]{-\frac{1}{2}-i\frac{\sqrt{3}}{2}}$,

$$a=-\frac{1}{2},\ b=-\frac{\sqrt{3}}{2},\ |z|=1,\ \operatorname{tg}\varphi=\sqrt{3}\implies\varphi=\frac{4\pi}{3}$$

$$w=\cos\frac{\frac{4\pi}{3}+2k\pi}{4}+i\sin\frac{\frac{4\pi}{3}+2k\pi}{4},\ k\in\mathbf{Z}_4$$

$$w=\cos\Big(\frac{\pi}{3}+k\frac{\pi}{2}\Big)+i\sin\Big(\frac{\pi}{3}+k\frac{\pi}{2}\Big),\ k\in\mathbf{Z}_4$$

$$w_1=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}=\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

$$w_2=\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}=-\frac{\sqrt{3}}{2}+\frac{1}{2}i$$

$$w_3=\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$$

$$w_4=\cos\frac{11\pi}{6}+i\sin\frac{11\pi}{6}=\frac{\sqrt{3}}{2}-\frac{1}{2}i$$

2) $\sqrt[3]{2+2i}$,

$$a=2,\ b=2,\ |z|=\sqrt{8},\ \varphi=\frac{\pi}{4}$$

$$w=\sqrt{2}\left(\cos\frac{\frac{\pi}{4}+2k\pi}{3}+i\sin\frac{\frac{\pi}{4}+2k\pi}{3}\right),\ k\in\mathbf{Z}_3$$

$$w=\sqrt{2}\Big(\cos\frac{1+8k}{12}\pi+i\sin\frac{1+8k}{12}\pi\Big),\ k\in\mathbf{Z}_3$$

$$w_1=\sqrt{2}\Big(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\Big)=\sqrt{2}\Big[\cos\Big(\frac{\pi}{3}-\frac{\pi}{4}\Big)+i\sin\Big(\frac{\pi}{3}-\frac{\pi}{4}\Big)\Big]$$

$$=\sqrt{2}\Big(\cos\frac{\pi}{3}\cos\frac{\pi}{4}+\sin\frac{\pi}{3}\sin\frac{\pi}{4}+\Big(\sin\frac{\pi}{3}\cos\frac{\pi}{4}-\cos\frac{\pi}{3}\sin\frac{\pi}{4}\Big)i\Big)$$

$$=\sqrt{2}\Big(\frac{1}{2}\cdot\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}+\Big(\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}\Big)i\Big)$$

$$=\sqrt{2}\frac{\sqrt{2}}{2}\Big(\frac{1}{2}+\frac{\sqrt{3}}{2}+\Big(\frac{\sqrt{3}}{2}-\frac{1}{2}\Big)i\Big)=\frac{\sqrt{3}+1}{2}+\frac{\sqrt{3}-1}{2}i$$

$$w_2=\sqrt{2}\Big(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\Big)=\sqrt{2}\Big(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\Big)=-1+i$$

$$w_3=\sqrt{2}\Big(\cos\frac{17\pi}{12}+i\sin\frac{17\pi}{12}\Big)=-\sqrt{2}\Big(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\Big)$$

$$=-\sqrt{2}\Big[\cos\Big(\frac{3\pi}{4}-\frac{\pi}{3}\Big)+i\sin\Big(\frac{3\pi}{4}-\frac{\pi}{3}\Big)\Big]$$

$$= -\sqrt{2}\left(\cos\frac{3\pi}{4}\cos\frac{\pi}{3} + \sin\frac{3\pi}{4}\sin\frac{\pi}{3} + i\left(\sin\frac{3\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{3}\cos\frac{3\pi}{4}\right)\right)$$

$$= -\sqrt{2}\left[-\frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \left(\frac{\sqrt{2}}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}\right)i\right]$$

$$= -\sqrt{2}\frac{\sqrt{2}}{2}\left[-\frac{1}{2} + \frac{\sqrt{3}}{2} + \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)i\right] = \frac{1-\sqrt{3}}{2} - \frac{1+\sqrt{3}}{2}i$$

3) $\sqrt[6]{\dfrac{1-i}{\sqrt{3}+i}}$,

$$\frac{1-i}{\sqrt{3}+i} = \frac{\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)}{2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)} = \frac{\sqrt{2}}{2}\left(\cos\left(\frac{7\pi}{4} - \frac{\pi}{6}\right) + i\sin\left(\frac{7\pi}{4} - \frac{\pi}{6}\right)\right)$$

$$= \frac{1}{\sqrt{2}}\left(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right)$$

$$w = \sqrt[6]{\frac{1-i}{\sqrt{3}+i}} = \frac{1}{\sqrt[12]{2}}\left(\cos\frac{\frac{19\pi}{12} + 2k\pi}{6} + i\sin\frac{\frac{19\pi}{12} + 2k\pi}{6}\right)$$

$$w_k = \frac{1}{\sqrt[12]{2}}\left(\cos\frac{24k+19}{72}\pi + i\sin\frac{24k+19}{72}\pi\right),\ k \in \mathbf{Z}_6$$