

Zadatak 17. Izračunaj:

$$1) \sqrt[8]{\frac{1+i}{\sqrt{3}-i}}; \quad 2) \sqrt[6]{\frac{1-i}{1+i\sqrt{3}}}; \quad 3) \sqrt[12]{\frac{1}{1-i\sqrt{3}}}.$$

Rješenje. 1) $\sqrt[8]{\frac{1+i}{\sqrt{3}-i}},$

$$\begin{aligned} \frac{1+i}{\sqrt{3}-i} &= \frac{\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)}{2\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)} = \frac{1}{\sqrt{2}}\left[\cos\left(\frac{\pi}{4} - \frac{11\pi}{6}\right) + i\sin\left(\frac{\pi}{4} - \frac{11\pi}{6}\right)\right] \\ &= \frac{1}{\sqrt{2}}\left(\cos\left(-\frac{19\pi}{12}\right) + i\sin\left(-\frac{19\pi}{12}\right)\right) = \frac{1}{\sqrt{2}}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right) \end{aligned}$$

$$w = \frac{1}{\sqrt[8]{2}}\left(\cos\frac{\frac{5\pi}{12} + 2k\pi}{8} + i\sin\frac{\frac{5\pi}{12} + 2k\pi}{8}\right), \quad k \in \mathbf{Z}_8$$

$$w_k = \frac{1}{\sqrt[8]{2}}\left(\cos\frac{24k+5}{96}\pi + i\sin\frac{24k+5}{96}\pi\right), \quad k \in \mathbf{Z}_8$$

2) $\sqrt[6]{\frac{1-i}{1+i\sqrt{3}}},$

$$\begin{aligned} \frac{1-i}{1+i\sqrt{3}} &= \frac{\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)}{2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)} = \frac{1}{\sqrt{2}}\left[\cos\left(\frac{7\pi}{4} - \frac{\pi}{3}\right) + i\sin\left(\frac{7\pi}{4} - \frac{\pi}{3}\right)\right] \\ &= \frac{1}{\sqrt{2}}\left(\cos\frac{17\pi}{12} + i\sin\frac{17\pi}{12}\right) \end{aligned}$$

$$w = \frac{1}{\sqrt[6]{2}}\left[\cos\frac{\frac{17\pi}{12} + 2k\pi}{6} + i\sin\frac{\frac{17\pi}{12} + 2k\pi}{6}\right], \quad k \in \mathbf{Z}_6$$

$$w_k = \frac{1}{\sqrt[6]{2}}\left(\cos\frac{24k+17}{72}\pi + i\sin\frac{24k+17}{72}\pi\right), \quad k \in \mathbf{Z}_6$$

3) $\sqrt[12]{\frac{1}{1-i\sqrt{3}}},$

$$\frac{1}{1-i\sqrt{3}} = \frac{\cos 0 + i\sin 0}{2\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right)} = \frac{1}{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$w = \frac{1}{\sqrt[12]{2}}\left(\cos\frac{\frac{\pi}{3} + 2k\pi}{12} + i\sin\frac{\frac{\pi}{3} + 2k\pi}{12}\right), \quad k \in \mathbf{Z}_{12}$$

$$w_k = \frac{1}{\sqrt[12]{2}}\left(\cos\frac{6k+1}{36}\pi + i\sin\frac{6k+1}{36}\pi\right), \quad k \in \mathbf{Z}_{12}$$