Zadatak 18. Odredi sve kompleksne brojeve z takve da je:

1) $z^4 = \cos\frac{2\pi}{3} - i\sin\frac{\pi}{3}$;

2) $z^3 = -\cos\frac{\pi}{4} + i\sin\frac{3\pi}{4}$.

Rješenje. **1)** $z^4 = \cos\frac{2\pi}{3} - i\sin\frac{\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i \implies z = \sqrt[4]{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}$;

$$a = -\frac{1}{2},\ b = -\frac{\sqrt{3}}{2},\ |z| = 1,\ \operatorname{tg}\varphi = \sqrt{3} \implies \varphi = \frac{4\pi}{3}$$

$$w = \cos\frac{\frac{4\pi}{3} + 2k\pi}{4} + i\sin\frac{\frac{4\pi}{3} + 2k\pi}{4},\ k \in \mathbf{Z}_4$$

$$w = \cos\left(\frac{\pi}{3} + k\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{3} + k\frac{\pi}{2}\right),\ k \in \mathbf{Z}_4$$

$$w_1 = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$w_2 = \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$$

$$w_3 = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$w_4 = \cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6} = \frac{\sqrt{3}}{2} - \frac{1}{2}i$$

2) $z^3 = -\cos\frac{\pi}{4} + i\sin\frac{3\pi}{4} = \cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}$.

$$z = \cos\frac{\frac{3\pi}{4} + 2k\pi}{3} + i\sin\frac{\frac{3\pi}{4} + 2k\pi}{3}, \quad k = 0, 1, 2$$

$$= \cos\frac{3\pi + 8k\pi}{12} + i\sin\frac{3\pi + 8k\pi}{12}$$

$$z_0 = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$$

$$z_1 = \cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}$$

$$z_2 = \cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}$$