

Zadatak 20. Prikaži u trigonometrijskom obliku kompleksne brojeve $z = -\sqrt{3} - i$ i $w = \cos \frac{2\pi}{3} - i \sin \frac{4\pi}{3}$.

Izračunaj $\sqrt[3]{z}$ i w^5 .

Rješenje.

$$z = -\sqrt{3} - i,$$

$$r = \sqrt{1+3} = 2;$$

$$\left. \begin{array}{l} 2 \cos \varphi = -\sqrt{3}, \quad \cos \varphi = -\frac{\sqrt{3}}{2}; \\ 2 \sin \varphi = -1, \quad \sin \varphi = -\frac{1}{2}. \end{array} \right\} \Rightarrow \varphi = \frac{7\pi}{6};$$

$$z = 2 \left(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right);$$

$$\begin{aligned} \sqrt[3]{z} &= \sqrt[3]{2} \left(\cos \frac{\frac{7\pi}{6} + 2k\pi}{3} + i \sin \frac{\frac{7\pi}{6} + 2k\pi}{3} \right) \\ &= \sqrt[3]{2} \left(\cos \frac{7\pi + 12k\pi}{18} + i \sin \frac{7\pi + 12k\pi}{18} \right) \quad k = 0, 1, 2; \end{aligned}$$

$$z_0 = \sqrt[3]{2} \left(\cos \frac{7\pi}{18} + i \sin \frac{7\pi}{18} \right);$$

$$z_1 = \sqrt[3]{2} \left(\cos \frac{19\pi}{18} + i \sin \frac{19\pi}{18} \right);$$

$$z_2 = \sqrt[3]{2} \left(\cos \frac{31\pi}{18} + i \sin \frac{31\pi}{18} \right);$$

$$w = \cos \frac{2\pi}{3} - i \sin \frac{4\pi}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i;$$

$$\left. \begin{array}{l} r = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1; \\ \cos \varphi = -\frac{1}{2}; \\ \sin \varphi = \frac{\sqrt{3}}{2} \end{array} \right\} \Rightarrow \varphi = \frac{2\pi}{3};$$

$$w = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}.$$

$$w^5 = 5 \left(\cos \frac{10\pi}{3} + i \sin \frac{10\pi}{3} \right) = 5 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3} \right).$$