

Zadatak 21. Ako je $z = -4 \cos \frac{5\pi}{6} + 4i \sin \frac{7\pi}{6}$, izračunaj \bar{z}^3 i $\sqrt[4]{z}$.

Rješenje.

$$z = -4 \cos \frac{5\pi}{6} + 4i \sin \frac{7\pi}{6} = 2\sqrt{3} - 2i,$$

$$r = \sqrt{12 + 4} = 4;$$

$$\left. \begin{array}{l} 4 \cos \varphi = 2\sqrt{3}, \quad \cos \varphi = \frac{\sqrt{3}}{2} \\ 4 \sin \varphi = -2, \quad \sin \varphi = -\frac{1}{2} \end{array} \right\} \Rightarrow \varphi = \frac{11\pi}{6};$$

$$z = 4 \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6} \right);$$

$$\begin{aligned} \sqrt[4]{z} &= \sqrt[4]{4} \left(\cos \frac{\frac{11\pi}{6} + 2k\pi}{4} + i \sin \frac{\frac{11\pi}{6} + 2k\pi}{4} \right) \\ &= \sqrt{2} \left(\cos \frac{11\pi + 12k\pi}{24} + i \sin \frac{11\pi + 12k\pi}{24} \right), \quad k = 0, 1, 2, 3 \end{aligned}$$

$$z_0 = \sqrt{2} \left(\cos \frac{11\pi}{24} + i \sin \frac{11\pi}{24} \right)$$

$$z_1 = \sqrt{2} \left(\cos \frac{23\pi}{24} + i \sin \frac{23\pi}{24} \right)$$

$$z_2 = \sqrt{2} \left(\cos \frac{35\pi}{24} + i \sin \frac{35\pi}{24} \right)$$

$$z_3 = \sqrt{2} \left(\cos \frac{47\pi}{24} + i \sin \frac{47\pi}{24} \right);$$

$$\bar{z} = 2\sqrt{3} + 2i; \quad \left. \begin{array}{l} 4 \cos \bar{\varphi} = 2\sqrt{3}, \quad \cos \bar{\varphi} = \frac{\sqrt{3}}{2} \\ 4 \sin \bar{\varphi} = 2, \quad \sin \bar{\varphi} = \frac{1}{2} \end{array} \right\} \Rightarrow \bar{\varphi} = \frac{\pi}{6};$$

$$\bar{z} = 4 \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right);$$

$$\bar{z}^3 = 4^3 \left(\cos \frac{3\pi}{6} + i \sin \frac{3\pi}{6} \right) = 64 \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right).$$