

**Zadatak 22.** Ako je  $z = -\cos \frac{2\pi}{3} + i \sin \frac{4\pi}{3}$ , izračunaj  $z^4$  i  $\sqrt[5]{z}$ .

$$Rješenje. \quad z = -\cos \frac{2\pi}{3} + i \sin \frac{4\pi}{3} = \frac{1}{2} - \frac{\sqrt{3}}{2}i,$$

$$\left. \begin{aligned} r &= \sqrt{\frac{1}{4} + \frac{3}{4}} = 1; & \cos \varphi &= \frac{1}{2}; \\ && \sin \varphi &= -\frac{\sqrt{3}}{2}; \end{aligned} \right\} \Rightarrow \varphi = \frac{5\pi}{3};$$

$$z = \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3};$$

$$z^4 = \cos \frac{20\pi}{3} + i \sin \frac{20\pi}{3} = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3};$$

$$\left. \begin{aligned} \bar{z} &= \frac{1}{2} + \frac{\sqrt{3}}{2}i; & \cos \bar{\varphi} &= \frac{1}{2}; \\ && \sin \bar{\varphi} &= \frac{\sqrt{3}}{2}; \end{aligned} \right\} \Rightarrow \bar{\varphi} = \frac{\pi}{3};$$

$$\bar{z} = \cos \frac{\pi}{3} + i \sin \frac{\pi}{3};$$

$$\begin{aligned} \sqrt[5]{z} &= \cos \frac{\frac{\pi}{3} + 2k\pi}{5} + i \sin \frac{\frac{\pi}{3} + 2k\pi}{5} \\ &= \cos \frac{\pi + 6k\pi}{15} + i \sin \frac{\pi + 6k\pi}{15}, \quad k = 0, 1, 2, 3, 4 \end{aligned}$$

$$\bar{z}_0 = \sqrt{2} \left( \cos \frac{\pi}{15} + i \sin \frac{\pi}{15} \right)$$

$$\bar{z}_1 = \sqrt{2} \left( \cos \frac{7\pi}{15} + i \sin \frac{7\pi}{15} \right)$$

$$\bar{z}_2 = \sqrt{2} \left( \cos \frac{13\pi}{15} + i \sin \frac{13\pi}{15} \right)$$

$$\bar{z}_3 = \sqrt{2} \left( \cos \frac{19\pi}{15} + i \sin \frac{19\pi}{15} \right)$$

$$\bar{z}_4 = \sqrt{2} \left( \cos \frac{25\pi}{15} + i \sin \frac{25\pi}{15} \right) = \sqrt{2} \left( \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right).$$