

Zadatak 23. Odredi ona rješenja jednačbe $z^7 + \bar{z} = 0$, $z \in \mathbf{C}$ za koja je $\text{Im}(z) > 0$.

Rješenje.

$$\begin{aligned} z^7 + \bar{z} = 0, z \in \mathbf{C}, \text{Im}(z) > 0; z^7 = -\bar{z}, |z^7| = |z|^7 = |-\bar{z}| = |z| \implies z = 0 \\ \text{ili } |z| = 1; z^8 = -|z|^2 = -1 \implies z = \sqrt[8]{-1}; 1.212.10 \implies z_k = \\ \cos \frac{2k+1}{8}\pi + i \sin \frac{2k+1}{8}\pi, k \in \mathbf{Z}_8; \text{Im} z > 0 \implies \sin \frac{2k+1}{8}\pi > 0, \\ k \in \mathbf{Z}_8 \implies \frac{2k+1}{8}\pi < \pi, k \in \mathbf{Z}_8 \implies \frac{2k+1}{8} < 1, k \in \mathbf{Z}_8 \implies z_k = \\ \cos \frac{2k+1}{8}\pi + i \sin \frac{2k+1}{8}\pi, k \in \mathbf{Z}_k. \end{aligned}$$