

Zadatak 24. Pokaži da vrijedi

$$\begin{aligned} & \left(\frac{-1+i\sqrt{3}}{2}\right)^n + \left(\frac{-1-i\sqrt{3}}{2}\right)^n \\ &= \begin{cases} 2, & \text{ako je } n = 3k, \\ -1, & \text{ako je } n = 3k \pm 1. \end{cases} \end{aligned}$$

Rješenje.

$$\begin{aligned} \left(\frac{-1+i\sqrt{3}}{2}\right)^3 &= \frac{-3i\sqrt{3}+9+3i\sqrt{3}-1}{8} = 1 \\ \left(\frac{-1-i\sqrt{3}}{2}\right)^3 &= -\frac{1}{8}(1+3i\sqrt{3}-9-3i\sqrt{3}) = 1 \\ \left(\frac{-1+i\sqrt{3}}{2}\right)^{3k} + \left(\frac{-1-i\sqrt{3}}{2}\right)^{3k} &= \left[\left(\frac{-1+i\sqrt{3}}{2}\right)^3\right]^k + \left[\left(\frac{-1-i\sqrt{3}}{2}\right)^3\right]^k \\ &= 1^k + 1^k = 2 \\ \left(\frac{-1+i\sqrt{3}}{2}\right)^{3k+1} + \left(\frac{-1-i\sqrt{3}}{2}\right)^{3k+1} &= 1^k \cdot \frac{-1+i\sqrt{3}}{2} + 1^k \cdot \frac{-1-i\sqrt{3}}{2} \\ &= \frac{-1+i\sqrt{3}-1-i\sqrt{3}}{2} = -1 \end{aligned}$$