**Zadatak 25.** Riješi jednadžbe:

**1)** $z^6 + 64 = 0$;

**2)** $z^4 + 1 = 0$;

**3)** $z^5+z^4+z^3+z^2+z+1=0$;

**4)** $z^3 + i = 0$;

**5)** $z^5 - i = 0$;

**6)** $z^6 - 1 = 0$.

***Rješenje.*** **1)** $z^6 + 64 = 0$,

$$z_k = \sqrt[6]{-64} = 2\left(\cos\frac{\pi + 2k\pi}{6} + i\sin\frac{\pi + 2k\pi}{6}\right),\ k = 0,1,2,3,4,5$$

$$z_0 = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \sqrt{3} + i$$

$$z_1 = 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = 2i$$

$$z_2 = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) = 2\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = -\sqrt{3} + i$$

$$z_3 = 2\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{2}\right) = -\sqrt{3} - i$$

$$z_4 = 2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) = -2i$$

$$z_5 = 2\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right) = \sqrt{3} - i$$

**2)** $z^4 + 1 = 0$, $z^4 = -1 \implies z = \sqrt[4]{-1}$;

$$z_k = \cos\frac{\pi + 2k\pi}{4} + i\sin\frac{\pi + 2k\pi}{4},\ k = 0,1,2,3$$

$$z_0 = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$$

$$z_1 = \cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4} = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$$

$$z_2 = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$$

$$z_3 = \cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$$

**3)** $z^5 + z^4 + z^3 + z^2 + z + 1 = 0 \ / \ \cdot (z-1)$; $\quad z = \sqrt[6]{1}$

$$z_k = \cos\frac{2k\pi}{6} + i\sin\frac{2k\pi}{6},\ k = 0,1,2,3,4,5$$
$$z_0 = \cos 0 + i\sin 0 = 1$$
$$z_1 = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$
$$z_2 = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$
$$z_3 = \cos\pi + i\sin\pi = -1$$
$$z_4 = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$
$$z_5 = \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

**4)** $z^3 + i = 0$,

$$z^3 = -i \implies z = \sqrt[3]{-i}$$
$$z_k = \cos\frac{\frac{3\pi}{2} + 2k\pi}{3} + i\sin\frac{\frac{3\pi}{2} + 2k\pi}{3}$$
$$= \cos\frac{3\pi + 4k\pi}{6} + i\sin\frac{3\pi + 4k\pi}{6},\ k = 0,1,2$$
$$z_0 = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$$
$$z_1 = \cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6} = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$$
$$z_2 = \cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6} = \frac{\sqrt{3}}{2} - \frac{1}{2}i$$

**5)** $z^5 - i = 0,\ z = \sqrt[5]{i}$

$$z_k = \cos\frac{\frac{\pi}{2} + 2k\pi}{5} + i\sin\frac{\frac{\pi}{2} + 2k\pi}{5}$$
$$= \cos\frac{\pi + 4k\pi}{10} + i\sin\frac{\pi + 4k\pi}{10},\ k = 0,1,2,3,4$$
$$z_0 = \cos\frac{\pi}{10} + i\sin\frac{\pi}{10};$$
$$z_1 = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i;$$
$$z_2 = \cos\frac{9\pi}{10} + i\sin\frac{9\pi}{10};$$
$$z_3 = \cos\frac{13\pi}{10} + i\sin\frac{13\pi}{10};$$
$$z_4 = \cos\frac{17\pi}{10} + i\sin\frac{17\pi}{10}.$$

**6)** $z^6 - 1 = 0,\ z^6 = 1 \implies z = \sqrt[6]{1};$

$$z_k = \cos\frac{2k\pi}{6} + i\sin\frac{2k\pi}{6},\ k = 0, 1, 2, 3, 4, 5$$

$$z_0 = \cos 0 + i\sin 0 = 1$$

$$z_1 = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$z_2 = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$z_3 = \cos\pi + i\sin\pi = -1$$

$$z_4 = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$z_5 = \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$$