

Zadatak 49. Dokaži: $\left(\frac{1+i \operatorname{tg} \alpha}{1-i \operatorname{tg} \alpha}\right)^n = \frac{1+i \operatorname{tg} n\alpha}{1-\operatorname{tg} n\alpha}$.

Rješenje. Uvrstiti ćemo $\operatorname{tg} \alpha = \sin \alpha / \cos \alpha$, srediti i primjeniti de Moivreove formulu na brojnik i nazivnik:

$$\begin{aligned} \left(\frac{1+i \operatorname{tg} \alpha}{1-i \operatorname{tg} \alpha}\right)^n &= \left(\frac{1+i \frac{\sin \alpha}{\cos \alpha}}{1-i \frac{\sin \alpha}{\cos \alpha}}\right)^n = \left(\frac{\frac{\cos \alpha + i \sin \alpha}{\cos \alpha}}{\frac{\cos \alpha - i \sin \alpha}{\cos \alpha}}\right)^n \\ &= \left(\frac{\cos \alpha + i \sin \alpha}{\cos(-\alpha) + i \sin(-\alpha)}\right)^n = \frac{\cos(n\alpha) + i \sin(n\alpha)}{\cos(-n\alpha) + i \sin(-n\alpha)} \\ &= \frac{\cos(n\alpha) + i \sin(n\alpha)}{\cos(n\alpha) - i \sin(n\alpha)} = \frac{1+i \frac{\sin(n\alpha)}{\cos(n\alpha)}}{1-i \frac{\sin(n\alpha)}{\cos(n\alpha)}} = \frac{1+i \operatorname{tg}(n\alpha)}{1-i \operatorname{tg}(n\alpha)} \end{aligned}$$