Zadatak 50. Prikaži u trigonometrijskom obliku sljedeće brojeve:

1) $z = 1 - \sin\alpha + i\cos\alpha, \qquad (0 < \alpha < \frac{\pi}{2})$;

2) $z = \cos\alpha - i\sin\alpha, \qquad (\pi < \alpha < \frac{3\pi}{2})$;

3) $z = \dfrac{1+\cos\alpha + i\,\sin\alpha}{1+\cos\alpha - i\sin\alpha}, \qquad (0 < \alpha < \frac{\pi}{2})$.

Rješenje. **1)**

$$|z| = \sqrt{(1-\sin\alpha)^2 + \cos^2\alpha} = \sqrt{1 - 2\sin\alpha + 1} = \sqrt{2(1-\sin\alpha)}.$$

Da odredimo φ izračunajmo

$$\operatorname{tg}\varphi = \frac{\cos\alpha}{1-\sin\alpha} = \frac{\cos\alpha/2 + \sin\alpha/2}{\cos\alpha/2 - \sin\alpha/2} = \frac{1+\operatorname{tg}\alpha/2}{1-\operatorname{tg}\alpha/2} = \operatorname{tg}\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$$

te je $\varphi = \frac{\pi}{4} + \frac{\alpha}{2}$.

Trigonometrijski prikaz je

$$z = \sqrt{2\,(1-\sin\alpha)}[\cos\left(\frac{\pi}{4}+\frac{\alpha}{2}\right) + i\,\sin\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)].$$

2) z je modula 1.

$$z = \cos\alpha - i\sin\alpha = \cos(-\alpha) + i\sin(-\alpha) = \cos(2\pi - \alpha) + i\,\sin(2\pi-\alpha).$$

3) Računanjem algebarskoga prikaza broja z dobivamo:

$$\begin{aligned} z &= \frac{1+\cos\alpha + i\sin\alpha}{1+\cos\alpha - i\sin\alpha}\cdot\frac{1+\cos\alpha + i\sin\alpha}{1+\cos\alpha + i\sin\alpha} \\ &= \frac{1+\cos^2\alpha - \sin^2\alpha + 2\cos\alpha + 2i\sin\alpha + 2\cos\alpha\sin\alpha}{1+\cos^2\alpha + 2\cos\alpha + \sin^2\alpha} \\ &= \frac{2\cos^2\alpha + 2\cos\alpha + 2i\sin\alpha + 2\cos\alpha\sin\alpha}{2+2\cos\alpha} \\ &= \frac{\cos\alpha(2+2\cos\alpha) + i\sin\alpha(2+2\cos\alpha)}{2+2\cos\alpha} = \cos\alpha + i\sin\alpha. \end{aligned}$$