

**Zadatak 53.** Odredi sve kompleksne brojeve  $z$  za koje vrijedi:

- 1)  $|z + iz| = 2\sqrt{2}$ ,  $\operatorname{Re}(z^3) = 4\sqrt{3}$ ,  
 $\frac{3\pi}{2} \leq \arg z < 2\pi$ ;
- 2)  $|z| = 2$ ,  $\operatorname{Im}(z^6) = 8\operatorname{Im}(z^3)$ ;
- 3)  $\operatorname{Re}(z^4) - \frac{1}{2}\operatorname{Im}(z^4) = (\operatorname{Re}(z^2))^2$ ,  
 $\operatorname{Im}(z^2) = 2\sqrt{2}$ .

**Rješenje.** 1) Stavimo  $z = a + bi$ ,  $r = \sqrt{a^2 + b^2}$ . Pogledajmo što možemo zaključiti iz uvjeta zadatka:

$$|z + iz| = \sqrt{(a-b)^2 + (a+b)^2} = \sqrt{2}\sqrt{a^2 + b^2} = 2\sqrt{2}; \implies r = 2;$$

$$\operatorname{Re}(z^3) = 8 \cos(3\alpha) = 4\sqrt{3}; \implies \cos(3\alpha) = \frac{\sqrt{3}}{2}$$

$$\implies 3\alpha = \frac{\pi}{6} + 2k\pi \quad \text{ili} \quad 3\alpha = \frac{11\pi}{6} + 2k\pi$$

$$\implies \alpha = \frac{(1+12k)\pi}{18} \quad \text{ili} \quad \alpha = \frac{(11+12k)\pi}{18}$$

$$\alpha \in \left[ \frac{3\pi}{2}, 2\pi \right) = \left[ \frac{27\pi}{18}, \frac{36\pi}{18} \right) \implies \alpha = \frac{35}{18}\pi;$$

$$z = 2 \left( \cos \frac{35}{18}\pi + i \sin \frac{35}{18}\pi \right).$$

$$2) z_1 = 2 \left( \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right), z_2 = 2 \left( \cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3} \right), z_3 = 2 \left( \cos \frac{7\pi}{3} + i \sin \frac{7\pi}{3} \right);$$

$$3) z_1 = 2 \left( \cos \frac{3\pi}{8} + i \sin \frac{3\pi}{8} \right), z_2 = 2 \left( \cos \frac{11\pi}{8} + i \sin \frac{11\pi}{8} \right).$$