Rješenja zadataka 4.1

Zadatak 1. Izračunaj kvocijent $\frac{\Delta y}{\Delta x}$ za funkciju f u zadanoj točki x_0:

1) $f(x) = x^2 - 3x + 1$, $x_0 = 1$, $x_0 = 2$, u bilo kojoj točki $x_0 \in D_f$;

2) $f(x) = ax^2 + bx + c$, $x_0 = 1$, $x_0 = 2$, u bilo kojoj točki $x_0 \in D_f$;

3) $f(x) = \frac{1}{x}$, $x_0 = 1$, $x_0 = 2$, u bilo kojoj točki $x_0 \in D_f$;

4) $f(x) = \sqrt{x}$, $x_0 = 1$, $x_0 = 2$, u bilo kojoj točki $x_0 \in D_f$;

5) $f(x) = \frac{x-1}{x+1}$, $x_0 = 1$, $x_0 = 2$, u bilo kojoj točki $x_0 \in D_f$.

Rješenje. **1)** $f(x) = x^2 - 3x + 1$, $x_0 = 1$, $x_0 = 2$, $x_0 \in D_f$;

$$\begin{aligned}\Delta y &= f(x_0+\Delta x) - f(x_0) = (x_0+\Delta x)^2 - 3(x_0+\Delta x) + 1 - x_0^2 + 3x_0 - 1\\ &= x_0^2 + 2x_0\Delta x + \Delta x^2 - 3x_0 - 3\Delta x + 3x_0 - x_0^2 = \Delta x^2 + (2x_0 - 3)\Delta x\\ &\Longrightarrow \frac{\Delta y}{\Delta x} = \Delta x + 2x_0 - 3, \quad \frac{\Delta y}{\Delta x}\Big|_{x_0=1} = \Delta x - 1, \quad \frac{\Delta y}{\Delta x}\Big|_{x_0=2} = \Delta x + 1;\end{aligned}$$

2) $f(x) = ax^2 + bx + c$, $x_0 = 1$, $x_0 = 2$, $x_0 \in D_f$;

$$\begin{aligned}\Delta y &= f(x_0+\Delta x) - f(x_0) = a(x_0+\Delta x)^2 + b(x_0+\Delta x) + C - ax_0^2 - bx_0 - C\\ &= a(x_0^2 + 2x_0\Delta x + \Delta x^2) + bx_0 + b\Delta x - ax_0^2 - bx_0\\ &= ax_0^2 + 2ax_0\Delta x + a\Delta x^2 + b\Delta x - ax_0^2 = a\Delta x^2 + (2ax_0 + b)\Delta x\\ &\Longrightarrow \frac{\Delta y}{\Delta x} = a\Delta x + 2ax_0 + b, \quad \frac{\Delta y}{\Delta x}\Big|_{x_0=1} = a\Delta x + 2a + b, \quad \frac{\Delta y}{\Delta x}\Big|_{x_0=2} = a\Delta x + 4a + b;\end{aligned}$$

3) $f(x) = \frac{1}{x}$, $x_0 = 1$, $x_0 = 2$, $x_0 \in D_f$;

$$\begin{aligned}\Delta y &= f(x_0+\Delta x) - f(x_0) = \frac{1}{x_0+\Delta x} - \frac{1}{x_0} = \frac{x_0 - x_0 - \Delta x}{x_0(x_0+\Delta x)} = -\frac{\Delta x}{x_0(x_0+\Delta x)}\\ &\Longrightarrow \frac{\Delta y}{\Delta x} = -\frac{1}{x_0^2 + x_0\Delta x}, \quad \frac{\Delta y}{\Delta x}\Big|_{x_0=1} = -\frac{1}{1+\Delta x}, \quad \frac{\Delta y}{\Delta x}\Big|_{x_0=2} = -\frac{1}{4+2\Delta x};\end{aligned}$$

4) $f(x)=\sqrt{x}$, $x_0=1$, $x_0=2$, $x_0\in D_f$;

$$\Delta y=f(x_0+\Delta x)-f(x_0)=\sqrt{x_0+\Delta x}-\sqrt{x_0}=\frac{(\sqrt{x_0+\Delta x}-\sqrt{x_0})(\sqrt{x_0+\Delta x}+\sqrt{x_0})}{\sqrt{x_0+\Delta x}+\sqrt{x_0}}$$

$$=\frac{x_0+\Delta x-x_0}{\sqrt{x_0+\Delta x}+\sqrt{x_0}}=\frac{\Delta x}{\sqrt{x_0+\Delta x}+\sqrt{x_0}}$$

$$\Longrightarrow\ \frac{\Delta y}{\Delta x}=\frac{1}{\sqrt{x_0+\Delta x}+\sqrt{x_0}},\quad \frac{\Delta y}{\Delta x}\Big|_{x=1}=\frac{1}{\sqrt{1+\Delta x}+1},\quad \frac{\Delta y}{\Delta x}\Big|_{x=2}=\frac{1}{\sqrt{2+\Delta x}+4};$$

5) $f(x)=\dfrac{x-1}{x+1}$, $x_0=1$, $x_0=2$, $x_0\in D_f$;

$$\Delta y=f(x_0+\Delta x)-f(x_0)=\frac{x_0+\Delta x-1}{x_0+\Delta x+1}-\frac{x_0-1}{x_0+1}$$

$$=\frac{x_0^2+x_0\Delta x-x_0+x_0+\Delta x-1-(x_0^2+x_0\Delta x+x_0-x_0-\Delta x-1)}{(x_0+1)(x_0+\Delta x+1)}$$

$$=\frac{x_0^2+x_0\Delta x+\Delta x-1-x_0^2-x_0\Delta x+\Delta x+1}{(x_0+1)(x_0+\Delta x+1)}=\frac{2\Delta x}{(x_0+1)(x_0+\Delta x+1)}$$

$$\Longrightarrow\ \frac{\Delta y}{\Delta x}=\frac{2}{(x_0+1)(x_0+\Delta x+1)},\quad \frac{\Delta y}{\Delta x}\Big|_{x_0=1}=\frac{1}{2+\Delta x},\quad \frac{\Delta y}{\Delta x}\Big|_{x_0=2}=\frac{1}{2(3+\Delta x)}.$$