

Zadatak 2. Izračunaj $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x}$ za funkciju f u zadanoj točki x_0 .

1) $f(x) = \frac{1}{x}$, $x_0 = 1$, $x_0 = 2$, u bilo kojoj točki $x_0 \in D_f$;

2) $f(x) = \sqrt{x}$, $x_0 = 1$, $x_0 = 2$, u bilo kojoj točki $x_0 \in D_f$;

3) $f(x) = x^3$, $x_0 = 1$, $x_0 = 2$, u bilo kojoj točki $x_0 \in D_f$.

Rješenje. 1) $f(x) = \frac{1}{x}$;

$$\Delta y = f(x_0 + \Delta x) - f(x_0) = \frac{1}{x_0 + \Delta x} - \frac{1}{x_0} = \frac{x_0 - x_0 - \Delta x}{x_0(x_0 + \Delta x)} = -\frac{\Delta x}{x_0(x_0 + \Delta x)}$$

$$\frac{\Delta y}{\Delta x} = -\frac{1}{x_0(x_0 + \Delta x)}, \quad \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = -\frac{1}{x_0^2}, \quad \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} \Big|_{x_0=1} = -1,$$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} \Big|_{x_0=2} = -\frac{1}{4};$$

2) $f(x) = \sqrt{x}$;

$$\begin{aligned} \Delta y = f(x_0 + \Delta x) - f(x_0) &= \sqrt{x_0 + \Delta x} - \sqrt{x_0} \\ &= \frac{x_0 + \Delta x - x_0}{\sqrt{x_0 + \Delta x} + \sqrt{x_0}} = \frac{\Delta x}{\sqrt{x_0 + \Delta x} + \sqrt{x_0}} \end{aligned}$$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{1}{2\sqrt{x_0}}, \quad \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} \Big|_{x_0=1} = \frac{1}{2}, \quad \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} \Big|_{x_0=2} = \frac{1}{2\sqrt{2}};$$

3) $f(x) = x^3$;

$$\Delta y = f(x_0 + \Delta x) - f(x_0) = (x_0 + \Delta x)^3 - x_0^3 = x_0^3 + 3x_0^2\Delta x + 3x_0\Delta x^2 + \Delta x^3 - x_0^3$$

$$\frac{\Delta y}{\Delta x} = 3x_0^2 + 3x_0\Delta x + \Delta x^2$$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = 3x_0^2, \quad \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} \Big|_{x_0=1} = 3, \quad \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} \Big|_{x_0=2} = 12.$$