

Zadatak 10. U kojoj se točki sijeku tangente položene na parabolu $y = \frac{1}{2}(x-1)^2$ u njezinim točkama s apscisama -1 i 2 ?

Rješenje. $y = \frac{1}{2}(x-1)^2, x_0 = -1, x_0 = 2;$

$$y(-1) = 2, \quad y(2) = \frac{1}{2} \implies T_1(-1, 2), \quad T_2\left(2, \frac{1}{2}\right)$$

$$k = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{1}{\Delta x} \left[\frac{1}{2}(x_0 + \Delta x - 1)^2 - \frac{1}{2}(x_0 - 1)^2 \right]$$

$$k_1 = \lim_{\Delta x \rightarrow 0} \frac{1}{2\Delta x} [(\Delta x - 2)^2 - (-2)^2] = \lim_{\Delta x \rightarrow 0} \frac{1}{2\Delta x} [\Delta x^2 - 4\Delta x + 4 - 4]$$

$$= \lim_{\Delta x \rightarrow 0} \left(\frac{\Delta x}{2} - 2 \right) = -2$$

$$k_2 = \lim_{\Delta x \rightarrow 0} \frac{1}{2\Delta x} [(1 + \Delta x)^2 - (2 - 1)^2] = \lim_{\Delta x \rightarrow 0} \frac{1}{2\Delta x} [1 + 2\Delta x + \Delta x^2 - 1]$$

$$= \lim_{\Delta x \rightarrow 0} \left(1 + \frac{\Delta x}{2} \right) = 1$$

$$y = kx + l$$

$$2 = -2(-1) + l \implies l_1 = 0 \implies y = -2x$$

$$\frac{1}{2} = 1 \cdot 2 + l \implies l_2 = -\frac{3}{2} \implies y = x - \frac{3}{2}$$

$$2x + y = 0$$

$$\underline{x - y = \frac{3}{2}}$$

$$3x = \frac{3}{2} \implies x = \frac{1}{2}$$

$$2 \cdot \frac{1}{2} + y = 0 \implies y = -1 \implies T\left(\frac{1}{2}, -1\right)$$