

Zadatak 11. U kojoj točki parabole $y = x^2 - 2x + 5$ treba postaviti tangentu koja je okomita na pravac $x - y = 0$?

Rješenje.

$$y = x^2 - 2x + 5, \quad x - y = 0;$$

$$x - y = 0 \implies y = x \implies k_1 = 1 \implies k = -1$$

$$\begin{aligned} k &= \lim_{\Delta x \rightarrow 0} \frac{1}{\Delta x} [f(x_0 + \Delta x) - f(x_0)] \\ &= \lim_{\Delta x \rightarrow 0} \frac{1}{\Delta x} [(x_0 + \Delta x)^2 - 2(x_0 + \Delta x) + 5 - x_0^2 + 2x_0 - 5] \\ &= \lim_{\Delta x \rightarrow 0} \frac{1}{\Delta x} [x_0^2 + 2x_0\Delta x + \Delta x^2 - 2x_0 - 2\Delta x - x_0^2 + 2x_0] \\ &= \lim_{\Delta x \rightarrow 0} (\Delta x + 2x_0 - 2) = 2x_0 - 2 \end{aligned}$$

$$2x_0 - 2 = -1 \implies 2x_0 = 1 \implies x_0 = \frac{1}{2}, \quad y\left(\frac{1}{2}\right) = \frac{1}{4} - 1 + 5 = \frac{17}{4}$$

$$\implies T\left(\frac{1}{2}, \frac{17}{4}\right).$$