

Zadatak 4. 1) $f(x) = \frac{1}{x}$; 2) $f(x) = \frac{1+x}{x}$;

3) $f(x) = \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}$;

4) $f(x) = \frac{1+x+x^2+x^3}{x}$;

5) $f(x) = \frac{1+x+x^2+x^3}{x^3}$.

Rješenje. 1) $f'(x) = \left(\frac{1}{x}\right)' = -\frac{x'}{x^2} = -\frac{1}{x^2}$;

2) $f'(x) = \left(\frac{1+x}{x}\right)' = \left(\frac{1}{x} + 1\right)' = \left(\frac{1}{x}\right)' + 1' = -\frac{x'}{x^2} + 0 = -\frac{1}{x^2}$;

3) $f'(x) = \left(\frac{1}{x}\right)' + \left(\frac{1}{x^2}\right)' + \left(\frac{1}{x^3}\right)' = -\frac{1}{x^2} + (x^{-2})' + (x^{-3})' = -\frac{1}{x^2} - 2x^{-3} - 3x^{-4} = -\frac{1}{x^2} - \frac{2}{x^3} - \frac{3}{x^4}$;

4) $f'(x) = \left(\frac{1+x+x^2+x^3}{x}\right)' = \left(\frac{1}{x} + 1 + x + x^2\right)' = \left(\frac{1}{x}\right)' + 1' + x' + (x^2)' = -\frac{1}{x^2} + 0 + x^{1-1} + 2 \cdot x^{2-1} = 1 + 2x - \frac{1}{x^2}$;

5) $f'(x) = \left(\frac{1+x+x^2+x^3}{x^3}\right)' = \left(\frac{1}{x^3} + \frac{1}{x^2} + \frac{1}{x} + x\right)' = \left(\frac{1}{x^3}\right)' + \left(\frac{1}{x^2}\right)' + \left(\frac{1}{x}\right)' + 1' = (x^{-3})' + (x^{-2})' + (x^{-1})' + 0 = -3 \cdot x^{-3-1} - 2 \cdot x^{-2-1} - 1 \cdot x^{-1-1} = -3x^{-4} - 2x^{-3} - x^{-2} = -\frac{3}{x^4} - \frac{2}{x^3} - \frac{1}{x^2}$.