

Zadatak 5. Deriviraj sljedeće funkcije:

$$1) f(x) = \frac{2}{x} + \frac{3}{x^2};$$

$$2) f(x) = 3x^{\frac{2}{3}} - 4x^{\frac{5}{2}} + x^{-2};$$

$$3) f(x) = \frac{a}{\sqrt{x}} + \frac{b}{x\sqrt{x}};$$

$$4) f(x) = \frac{\pi}{x} + e^3;$$

$$5) f(x) = 4x^4 - 2\sqrt{x};$$

$$6) f(x) = \sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}.$$

$$7) f(x) = \sqrt{2x} + 2\sqrt[3]{x};$$

$$8) f(x) = \frac{3}{\sqrt{x}} + 3\sqrt{x}.$$

Rješenje.

$$1) f'(x) = \left(\frac{2}{x} + \frac{3}{x^2}\right)' = \left(\frac{2}{x}\right)' + \left(\frac{3}{x^2}\right)' = (2x^{-1})' + (3x^{-2})' = -1 \cdot 2x^{-1-1} - 2 \cdot 3x^{-2-1} = -2x^{-2} - 6x^{-3} = -\frac{2}{x^2} - \frac{6}{x^3};$$

$$2) f'(x) = (3x^{\frac{2}{3}} - 4x^{\frac{5}{2}} + x^{-2})' = (3x^{\frac{2}{3}})' - (4x^{\frac{5}{2}})' + (x^{-2})' = \frac{2}{3} \cdot 3x^{\frac{2}{3}-1} - \frac{5}{2} \cdot 4x^{\frac{5}{2}-1} - 2 \cdot x^{-2-1} = 2x^{-\frac{1}{3}} - 10x^{\frac{3}{2}} - 2x^{-3};$$

$$3) f'(x) = \left(\frac{a}{\sqrt{x}} + \frac{b}{x\sqrt{x}}\right)' = (ax^{-\frac{1}{2}} + bx^{-\frac{3}{2}})' = -\frac{1}{2} \cdot ax^{-\frac{1}{2}-1} - \frac{3}{2} \cdot bx^{-\frac{3}{2}-1} = -\frac{1}{2}ax^{-\frac{3}{2}} - \frac{3}{2}bx^{-\frac{5}{2}} = -\frac{a}{2x^{\frac{3}{2}}} - \frac{3b}{2x^{\frac{5}{2}}} = -\frac{a}{2x\sqrt{x}} - \frac{3b}{2x^2\sqrt{x}};$$

$$4) f'(x) = \left(\frac{\pi}{x} + e^3\right)' = \left(\frac{\pi}{x}\right)' + (e^3)' = (\pi x^{-1})' + 0 = -1 \cdot \pi x^{-1-1} = -\pi x^{-2} = -\frac{\pi}{x^2};$$

$$5) f'(x) = (4x^4 - 2\sqrt{x})' = (4x^4)' - (2\sqrt{x})' = 4 \cdot 4x^{4-1} - (2x^{\frac{1}{2}})' = 16x^3 - \frac{1}{2} \cdot 2x^{\frac{1}{2}-1} = 16x^3 - x^{-\frac{1}{2}} = 16x^3 - \frac{1}{x^{\frac{1}{2}}} = 16x^3 - \frac{1}{\sqrt{x}};$$

$$6) f'(x) = (\sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x})' = (x^{\frac{1}{2}} + x^{\frac{1}{3}} + x^{\frac{1}{4}})' = (x^{\frac{1}{2}})' + (x^{\frac{1}{3}})' + (x^{\frac{1}{4}})' = \frac{1}{2} \cdot x^{\frac{1}{2}-1} + \frac{1}{3} \cdot x^{\frac{1}{3}-1} + \frac{1}{4} \cdot x^{\frac{1}{4}-1} = \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{3}x^{-\frac{2}{3}} + \frac{1}{4}x^{-\frac{3}{4}} = \frac{1}{2x^{\frac{1}{2}}} + \frac{1}{3x^{\frac{2}{3}}} + \frac{1}{4x^{\frac{3}{4}}} = \frac{1}{2\sqrt{x}} + \frac{1}{3\sqrt[3]{x^2}} + \frac{1}{4\sqrt[4]{x^3}};$$

$$7) f'(x) = (\sqrt{2x} + 2\sqrt[3]{x})' = (\sqrt{2x})' + (2\sqrt[3]{x})' = (\sqrt{2}\sqrt{x})' + (2x^{\frac{1}{3}})' = (\sqrt{2}x^{\frac{1}{2}})' + \frac{2}{3} \cdot 2x^{\frac{1}{3}-1} = \frac{1}{2} \cdot \sqrt{2}x^{\frac{1}{2}-1} + \frac{2}{3}x^{-\frac{2}{3}} = \frac{\sqrt{2}}{2}x^{-\frac{1}{2}} + \frac{2}{3x^{\frac{2}{3}}} = \frac{\sqrt{2}}{2x^{\frac{1}{2}}} + \frac{2}{3\sqrt[3]{x^2}} = \frac{2}{3\sqrt[3]{x^2}} + \frac{\sqrt{2}}{2\sqrt{x}} + \frac{2}{3\sqrt[3]{x^2}};$$

$$\begin{aligned} \mathbf{8)} \quad f'(x) &= \left(\frac{3}{\sqrt{x}} + 3\sqrt{x} \right)' = \left(\frac{3}{\sqrt{x}} \right)' + (3\sqrt{x})' = \left(\frac{3}{x^{\frac{1}{2}}} \right)' + (3x^{\frac{1}{2}})' = \\ &= (3x^{-\frac{1}{2}})' + \frac{1}{2} \cdot 3x^{\frac{1}{2}-1} = -\frac{1}{2} \cdot 3x^{-\frac{1}{2}-1} + \frac{3}{2}x^{-\frac{1}{2}} = -\frac{3}{2}x^{-\frac{3}{2}} + \frac{3}{2x^{\frac{1}{2}}} = \\ &= -\frac{3}{2x^{\frac{3}{2}}} + \frac{3}{2\sqrt{x}} = -\frac{3}{2x\sqrt{x}} + \frac{3}{2\sqrt{x}}. \end{aligned}$$