

Zadatak 6. Izračunaj derivacije sljedećih funkcija u točki x_0 :

1) $f(x) = 1 - x^2$, $x_0 = 1$;

2) $f(x) = x^3 - x + 101$, $x_0 = 2$;

3) $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$, $x_0 = 4$;

4) $f(x) = \frac{1}{n+1}x^{n+1} - \frac{1}{n}$, $x_0 = -1$;

5) $f(x) = -\frac{1}{2}x^3 - \frac{1}{4}x^2 + x$, $x_0 = -2$.

Rješenje.

1) $f'(x) = (1 - x^2)' = 1' - (x^2)' = 0 - 2x^{2-1} = -2x$, $f'(1) = -2 \cdot 1 = -2$;

2) $f'(x) = (x^3 - x + 101)' = (x^3)' - x' + 101' = 3x^{3-1} - 1 + 0 = 3x^2 - 1$,
 $f'(2) = 3 \cdot 2^2 - 1 = 3 \cdot 4 - 1 = 12 - 1 = 11$;

3) $f'(x) = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)' = (\sqrt{x})' + \left(\frac{1}{\sqrt{x}}\right)' = (x^{\frac{1}{2}})' + \left(\frac{1}{x^{\frac{1}{2}}}\right)' =$
 $\frac{1}{2}x^{\frac{1}{2}-1} + (x^{-\frac{1}{2}})' = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}-1} = \frac{1}{2x^{\frac{1}{2}}} - \frac{1}{2}x^{-\frac{3}{2}} = \frac{1}{2} \left(\frac{1}{\sqrt{x}} - \frac{1}{x^{\frac{3}{2}}}\right) =$
 $\frac{1}{2} \left(\frac{1}{\sqrt{x}} - \frac{1}{x\sqrt{x}}\right)$, $f'(4) = \frac{1}{2} \left(\frac{1}{\sqrt{4}} - \frac{1}{4\sqrt{4}}\right) = \frac{1}{2} \left(\frac{1}{2} - \frac{1}{8}\right) = \frac{1}{2} \left(\frac{3}{8}\right) =$
 $\frac{3}{16}$;

4) $f'(x) = \left(\frac{1}{n+1}x^{n+1} - \frac{1}{n}\right)' = \left(\frac{1}{n+1}x^{n+1}\right)' - \left(\frac{1}{n}\right)' = (n+1) \cdot$
 $\frac{1}{n+1}x^{n+1-1} - 0 = x^n$, $f'(-1) = (-1)^n$;

5) $f'(x) = \left(-\frac{1}{2}x^3 - \frac{1}{4}x^2 + x\right)' = \left(-\frac{1}{2}x^3\right)' - \left(\frac{1}{4}x^2\right)' + x' = 3 \cdot$
 $\left(-\frac{1}{2}\right)x^{3-1} - 2 \cdot \frac{1}{4}x^{2-1} + 1 = -\frac{3}{2}x^2 - \frac{1}{2}x + 1$, $f'(-2) = -\frac{3}{2}(-2)^2 -$
 $\frac{1}{2}(-2) + 1 = -\frac{3}{2} \cdot 4 + 1 + 1 = -6 + 2 = -4$.