

Zadatak 8. Deriviraj sljedeće funkcije:

$$1) f(x) = \frac{5}{x-2};$$

$$2) f(x) = \frac{2x}{3-x};$$

$$3) f(x) = \frac{x+2}{x-2};$$

$$4) f(x) = \frac{3x+4}{2x-1};$$

$$5) f(x) = \frac{5x^2-1}{x+2};$$

$$6) f(x) = \frac{3x-1}{x^2-3};$$

$$7) f(x) = \frac{x^2-6x+5}{x-3};$$

$$8) f(x) = \frac{x^2-6x+8}{x^2-6x+9};$$

$$9) f(x) = \frac{2x^2+3x+1}{x+1};$$

$$10) f(x) = \frac{x^6+8}{x^4-2x^2+4};$$

$$11) f(x) = \frac{8x^3-10x^2+15x-27}{16x^4-x^2-18x-81};$$

$$12) f(x) = \frac{x^7-1}{x^7+1}.$$

Rješenje.

$$1) f'(x) = \left(\frac{5}{x-2} \right)' = \frac{5'(x-2) - 5(x-2)'}{(x-2)^2} = \frac{0 - 5(x' - 2')}{(x-2)^2} = -\frac{5(x^{1-1} - 0)}{(x-2)^2} = -\frac{5}{(x-2)^2};$$

$$2) f'(x) = \left(\frac{2x}{3-x} \right)' = \frac{(2x)'(3-x) - 2x(3-x)'}{(3-x)^2} = \frac{2x^{1-1}(3-x) - 2x(3' - x^{1-1})}{(3-x)^2} = \frac{2(3-x) - 2x(0-1)}{(3-x)^2} = \frac{6-2x+2x}{(3-x)^2} = \frac{6}{(3-x)^2};$$

$$3) f'(x) = \left(\frac{x+2}{x-2} \right)' = \frac{(x+2)'(x-2) - (x+2)(x-2)'}{(x-2)^2} = \frac{(x' + 2')(x-2) - (x+2)(x' - 2')}{(x-2)^2} = \frac{(x^{1-1} + 0)(x-2) - (x+2)(x^{1-1} - 0)}{(x-2)^2} = \frac{(x-2) - (x+2)}{(x-2)^2} = \frac{x-2-x-2}{(x-2)^2} = \frac{-4}{(x-2)^2} = \frac{4}{(x-2)^2};$$

$$4) f'(x) = \left(\frac{3x+4}{2x-1} \right)' = \frac{(3x+4)'(2x-1) - (3x+4)(2x-1)'}{(2x-1)^2} = \frac{[(3x)' + 4'](2x-1) - (3x+4)[(2x)' - 1']}{(2x-1)^2} = \frac{(3x^{1-1} + 0)(2x-1) - (3x+4)(2x^{1-1} - 0)}{(2x-1)^2} = \frac{3(2x-1) - 2(3x+4)}{(2x-1)^2} = \frac{6x-3-6x-8}{(2x-1)^2} = \frac{-11}{(2x-1)^2} = -\frac{11}{(2x-1)^2};$$

$$5) f'(x) = \left(\frac{5x^2-1}{x+2} \right)' = \frac{(5x^2-1)'(x+2) - (5x^2-1)(x+2)'}{(x+2)^2}$$

$$\begin{aligned}
 &= \frac{[(5x^2)' - 1'](x+2) - (5x^2 - 1)(x' + 2')}{(x+2)^2} \\
 &= \frac{(2 \cdot 5x^{2-1} - 0)(x+2) - (5x^2 - 1)(x^{1-1} + 0)}{(x+2)^2} = \frac{10x(x+2) - (5x^2 - 1)}{(x+2)^2} = \\
 &\frac{10x^2 + 20x - 5x^2 + 1}{(x+2)^2} = \frac{5x^2 + 21}{(x+2)^2};
 \end{aligned}$$

$$\begin{aligned}
 \text{6) } f'(x) &= \left(\frac{3x-1}{x^2-3} \right)' = \frac{(3x-1)'(x^2-3) - (3x-1)(x^2-3)'}{(x^2-3)^2} \\
 &= \frac{[(3x)' + 1'](x^2-3) - (3x-1)[(x^2)' - 3']}{(x^2-3)^2} \\
 &= \frac{(3x^{1-1} + 0)(x^2-3) - (3x-1)(2x^{2-1} - 0)}{(x^2-3)^2} = \frac{3(x^2-3) - (3x-1)(2x)}{(x^2-3)^2} = \\
 &\frac{3x^2 - 9 - 6x^2 + 2x}{(x^2-3)^2} = \frac{-3x^2 + 2x - 9}{(x^2-3)^2};
 \end{aligned}$$

$$\begin{aligned}
 \text{7) } f'(x) &= \left(\frac{x^2 - 6x + 5}{x-3} \right)' = \frac{(x^2 - 6x + 5)'(x-3) - (x^2 - 6x + 5)(x-3)'}{(x-3)^2} = \\
 &\frac{[(x^2)' - (6x)' + 5'](x-3) - (x^2 - 6x + 5)(x' - 3')}{(x-3)^2} \\
 &= \frac{(2x^{2-1} - 6x^{1-1} + 0)(x-3) - (x^2 - 6x + 5)(x^{1-1} - 0)}{(x-3)^2} \\
 &= \frac{(2x-6)(x-3) - (x^2 - 6x + 5)}{(x-3)^2} = \frac{2x^2 - 6x - 6x + 18 - x^2 - 6x - 5}{(x-3)^2} = \\
 &\frac{x^2 - 12x + 13}{(x-3)^2};
 \end{aligned}$$

$$\begin{aligned}
 \text{8) } f'(x) &= \left(\frac{x^2 - 6x + 8}{x^2 - 6x + 9} \right)' \\
 &= \frac{(x^2 - 6x + 8)'(x^2 - 6x + 9) - (x^2 - 6x + 8)(x^2 - 6x + 9)'}{(x^2 - 6x + 9)^2} \\
 &= \frac{[(x^2)' - (6x)' + 8'](x^2 - 6x + 9) - (x^2 - 6x + 8)[(x^2)' - (6x)' + 9']}{(x^2 - 6x + 9)^2} \\
 &= \frac{(2 \cdot x^{2-1} - 6x^{1-1} + 0)(x^2 - 6x + 9) - (x^2 - 6x + 8)(2 \cdot x^{2-1} - 6x^{1-1} + 0)}{(x^2 - 6x + 9)^2} = \\
 &\frac{(2x-6)(x^2 - 6x + 9) - (x^2 - 6x + 8)(2x-6)}{(x^2 - 6x + 9)^2} \\
 &= \frac{2(x-3)(x^2 - 6x + 9 - x^2 + 6x - 8)}{(x-3)^4} = \frac{2}{(x-3)^3};
 \end{aligned}$$

$$\begin{aligned}
 \text{9) } f'(x) &= \left(\frac{2x^2 + 3x + 1}{x+1} \right)' = \frac{(2x^2 + 3x + 1)'(x+1) - (2x^2 + 3x + 1)(x+1)'}{(x+1)^2} = \\
 &\frac{[(2x^2)' + (3x)' + 1'](x+1) - (2x^2 + 3x + 1)(x' + 1')}{(x+1)^2} \\
 &= \frac{(2 \cdot 2x^{2-1} + 3x^{1-1} + 0)(x+1) - [(2x+1)(x+1)](x^{1-1} + 0)}{(x+1)^2}
 \end{aligned}$$

$$= \frac{(4x+3)(x+1) - (2x+1)(x+1)}{(x+1)^2} = \frac{(x+1)(4x+3-2x-1)}{(x+1)^2} = \frac{2x+2}{x+1} = \frac{2(x+1)}{x+1} = 2;$$

$$\begin{aligned} 10) f'(x) &= \left(\frac{x^6+8}{x^4-2x^2+4} \right)' = \frac{(x^6+8)'(x^4-2x^2+4) - (x^6+8)(x^4-2x^2+4)'}{(x^4-2x^2+4)^2} \\ &= \frac{[(x^6)' + 8'](x^4-2x^2+4) - (x^6+8)[(x^4)' - (2x^2)' + 4'](x^4-2x^2+4)^2}{(x^4-2x^2+4)^2} \\ &= \frac{(6x^{6-1}+0)(x^4-2x^2+4) - (x^6+8)(4x^{4-1} - 2 \cdot 2x^{2-1} + 0)}{(x^4-2x^2+4)^2} \\ &= \frac{6x^5(x^4-2x^2+4) - (x^6+8)(4x^3-4x)}{(x^4-2x^2+4)^2} \\ &= \frac{6x^5(x^4-2x^2+4) - (x^2+2)(x^4-2x^2+4)(4x^3-4x)}{(x^4-2x^2+4)^2} \\ &= \frac{(x^4-2x^2+4)[6x^5 - (x^2+2)(4x^3-4x)]}{(x^4-2x^2+4)^2} = \frac{6x^5 - 4x^5 + 4x^3 - 8x^3 + 8x}{x^4-2x^2+4} = \\ &= \frac{2x^5 - 4x^3 + 8x}{x^4-2x^2+4} = \frac{2x(x^4-4x^2+2)}{x^4-2x^2+4} = 2x; \end{aligned}$$

$$\begin{aligned} 11) f'(x) &= \left(\frac{8x^3-10x^2+15x-27}{16x^2-x^2-18x-81} \right)' \\ &= \frac{(8x^3-10x^2+15x-27)'(16x^4-x^2-18x-81) - (8x^3-10x^2+15x-27)(16x^4-x^2-18x-81)'}{(16x^4-x^2-18x-81)^2} \\ &= \frac{[(8x^3)' - (10x^2)' + (15x)' - 27'] [16x^4 - (x^2+18x+81)^2] - [(2x)^3 - 3^3 - 5x(2x-3)] [(16x^4)' - (x^2)' - (18x)' - (81)']}{[16x^4 - (x^2+18x+81)^2]^2} \\ &= \frac{(3 \cdot 8x^{3-1} - 2 \cdot 10x^{2-1} + 15x^{1-1} - 0) [16x^4 - (x+9)^2] - [(2x-3)(4x^2+6x+9) - 5x(2x-3)] (4 \cdot 16x^{4-1} - 2 \cdot x^{2-1} - 18x^{1-1} - 0)}{[16x^4 - (x+9)^2]^2} \\ &= \frac{(24x^2-20x+15)[(4x^2-x-9)(4x^2+x+9)] - (2x-3)(4x^2+6x+9-5x)(64x^3-2x-18)}{[(4x^2-x-9)(4x^2+x+9)]^2} = \\ &= \frac{(4x^2+x+9)[(24x^2-20x+15)(4x^2-x-9) - (2x-3)(64x^3-2x-18)]}{(4x^2+x+9)^2(4x^2-x-9)^2} \\ &= \frac{(96x^4-24x^3-216x^2-80x^3+20x^2+180+60x^2-15x-135) - (128x^4-192x^3-4x^2+6x-36x+54)}{(4x^2+x+9)(4x^2-x-9)^2} \\ &= \frac{96x^4-104x^3-136x^2+165x-135-128x^4+192x^3+4x^2+30x-54}{(4x^2+x+9)(4x^2-x-9)^2} \\ &= \frac{-32x^4+88x^3-132x^2+195x-189}{(4x^2+x+9)(4x^2-x-9)^2} \\ &= \frac{-32x^4+96x^3-84x^2-8x^3-48x^2+195x-189}{(4x^2+x+9)(4x^4-x-9)^2} \end{aligned}$$

$$\begin{aligned}
&= \frac{4x^2(-8x^2 + 24x - 21) - 8x^3 + 24x^2 - 21x - 72x^2 + 216x - 189}{(4x^2 + x + 9)(4x^4 - x - 9)^2} \\
&= \frac{4x^2(-8x^2 + 24x - 21) + x(-8x^2 + 24x - 21) + 9(-8x^2 + 24x - 21)}{(4x^2 + x + 9)(4x^2 - x - 9)^2} \\
&= \frac{(4x^2 + x + 9)(-8x^2 + 24x - 21)}{(4x^2 + x + 9)(4x^2 - x - 9)^2} = \frac{-8x^2 + 24x - 21}{(4x^2 - x - 9)^2}; \\
\mathbf{11)} \quad f'(x) &= \left(\frac{x^7 - 1}{x^7 + 1} \right)' = \frac{(x^7 - 1)'(x^7 + 1) - (x^7 - 1)(x^7 + 1)'}{(x^7 + 1)^2} \\
&= \frac{[(x^7)' - 1'](x^7 + 1) - (x^7 - 1)[(x^7)' + 1']}{(x^7 + 1)^2} \\
&= \frac{(7 \cdot x^{7-1} - 0)(x^7 + 1) - (x^7 - 1)(7 \cdot x^{7-1} + 0)}{(x^7 + 1)^2} = \frac{7x^6(x^7 + 1) - 7x^6(x^7 - 1)}{(x^7 + 1)^2} = \\
&= \frac{7x^6(x^7 + 1 - x^7 + 1)}{(x^7 + 1)^2} = \frac{14x^6}{(x^7 + 1)^2}.
\end{aligned}$$