

**Zadatak 9.** Deriviraj sljedeće funkcije:

$$1) f(x) = \frac{x-1}{1+\sqrt{x}}; \quad 2) f(x) = \frac{x+1}{\sqrt[3]{x}+1};$$

$$3) f(x) = \frac{1-\sqrt[3]{x}}{\sqrt[3]{x}}; \quad 4) f(x) = \frac{\sqrt{x^3}-1}{\sqrt{x}-1};$$

$$5) f(x) = \frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x\sqrt{x}+x+\sqrt{x}};$$

$$6) f(x) = \frac{x\sqrt{x}-x}{x-\sqrt{x}};$$

$$7) f(x) = \frac{(1+\sqrt{x})^3}{x\sqrt{x}+2x+\sqrt{x}};$$

$$8) f(x) = \frac{(\sqrt{x}-x)(x\sqrt{x}+x+\sqrt{x})}{x\sqrt{x}-1}.$$

**Rješenje.**

$$1) f'(x) = \left( \frac{x-1}{1+\sqrt{x}} \right)' = \left( \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}+1} \right)' = (\sqrt{x}-1)' = (\sqrt{x})' - 1' = \frac{1}{2\sqrt{x}};$$

$$2) f'(x) = \left( \frac{x+1}{\sqrt[3]{x}+1} \right)' = \left( \frac{(\sqrt[3]{x}+1)(\sqrt[3]{x^2}-\sqrt[3]{x}+1)}{\sqrt[3]{x}+1} \right)' = (\sqrt[3]{x^2}-\sqrt[3]{x}+1)' = (\sqrt[3]{x^2})' - (\sqrt[3]{x})' + 1' = (x^{\frac{2}{3}})' - (x^{\frac{1}{3}})' + 0 = \frac{2}{3}x^{\frac{2}{3}-1} - \frac{1}{3}x^{\frac{1}{3}-1} = \frac{2}{3}x^{-\frac{1}{3}} - \frac{1}{3}x^{-\frac{2}{3}} = \frac{2}{3x^{\frac{1}{3}}} - \frac{1}{3x^{\frac{2}{3}}} = \frac{2}{3\sqrt[3]{x}} - \frac{1}{3\sqrt[3]{x^2}};$$

$$3) f'(x) = \left( \frac{1-\sqrt[3]{x}}{\sqrt[3]{x}} \right)' = \left( \frac{1}{\sqrt[3]{x}} - 1 \right)' = \left( \frac{1}{x^{\frac{1}{3}}} \right)' - 1' = (x^{-\frac{1}{3}})' - 0 = -\frac{1}{3}x^{-\frac{1}{3}-1} = -\frac{1}{3}x^{-\frac{4}{3}} = -\frac{1}{3x^{\frac{4}{3}}} = -\frac{1}{3x\sqrt[3]{x}};$$

$$4) f'(x) = \left( \frac{\sqrt{x^3}-1}{\sqrt{x}-1} \right)' = \left( \frac{(\sqrt{x}-1)(x+\sqrt{x}+1)}{\sqrt{x}-1} \right)' = (x+\sqrt{x}+1)' = x' + (\sqrt{x})' + 1' = x^{1-1} + (x^{\frac{1}{2}})' + 0 = 1 + \frac{1}{2}x^{\frac{1}{2}-1} = 1 + \frac{1}{2}x^{-\frac{1}{2}} = 1 + \frac{1}{2x^{\frac{1}{2}}} = 1 + \frac{1}{2\sqrt{x}};$$

$$5) f'(x) = \left( \frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x\sqrt{x}+x+\sqrt{x}} \right)' = \left( \frac{\sqrt{x}(\sqrt{x}+1)(x\sqrt{x}-1)}{\sqrt{x}(x+\sqrt{x}+1)} \right)' = \left( \frac{(\sqrt{x}+1)(\sqrt{x}-1)(x+\sqrt{x}+1)}{x+\sqrt{x}+1} \right)' = (x-1)' = x' - 1' = x^{1-1} - 0 = 1;$$

$$6) f'(x) = \left( \frac{x\sqrt{x}-x}{x-\sqrt{x}} \right)' = \left( \frac{x(\sqrt{x}-1)}{\sqrt{x}(\sqrt{x}-1)} \right)' = (\sqrt{x})' = \frac{1}{2\sqrt{x}};$$

$$7) f'(x) = \left( \frac{(1+\sqrt{x})^3}{x\sqrt{x}+2x+\sqrt{x}} \right)' = \left( \frac{(1+\sqrt{x})^3}{\sqrt{x}(x+2\sqrt{x}+1)} \right)' = \left( \frac{(1+\sqrt{x})^3}{\sqrt{x}(1+\sqrt{x})^2} \right)' =$$

$$\left(\frac{1+\sqrt{x}}{\sqrt{x}}\right)' = \left(\frac{1}{\sqrt{x}}\right)' + 1' = (x^{-\frac{1}{2}})' = -\frac{1}{2}x^{-\frac{1}{2}-1} = -\frac{1}{2}x^{-\frac{3}{2}} = -\frac{1}{2x^{\frac{3}{2}}} = -\frac{1}{2x\sqrt{x}};$$

$$\mathbf{8)} f'(x) = \left(\frac{(\sqrt{x}-x)(x\sqrt{x}+x+\sqrt{x})}{x\sqrt{x}-1}\right)' = \left(\frac{\sqrt{x}(1-\sqrt{x})\sqrt{x}(x+\sqrt{x}+1)}{(\sqrt{x}-1)(x+\sqrt{x}+1)}\right)' = -(x)' = -x^{1-1} = -1.$$