

- Zadatak 10.**
- 1) $f(x) = \frac{1}{\sin x}$; 2) $f(x) = \frac{1}{\sin x} + \frac{1}{\cos x}$;
 3) $f(x) = \frac{1}{\operatorname{tg} x}$; 4) $f(x) = \frac{\operatorname{tg} x}{\operatorname{ctg} x}$;
 5) $f(x) = \operatorname{tg} x \cdot \operatorname{ctg} x$;
 6) $f(x) = \operatorname{tg} x + \operatorname{ctg} x$.

Rješenje.

$$1) f'(x) = \left(\frac{1}{\sin x} \right)' = \frac{1'(\sin x) - 1 \cdot (\sin x)'}{\sin^2 x} = -\frac{\cos x}{\sin^2 x};$$

$$2) f'(x) = \left(\frac{1}{\sin x} + \frac{1}{\cos x} \right)' = \left(\frac{1}{\sin x} \right)' + \left(\frac{1}{\cos x} \right)' = -\frac{\cos x}{\sin^2 x} + \frac{\sin x}{\cos^2 x} = \frac{\sin^3 x - \cos^3 x}{\sin^2 x \cdot \cos^2 x};$$

$$3) f'(x) = \left(\frac{1}{\operatorname{tg} x} \right)' = \frac{1' \cdot \operatorname{tg} x - 1 \cdot (\operatorname{tg} x)'}{\operatorname{tg}^2 x} = -\frac{\frac{1}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}} = -\frac{1}{\sin^2 x};$$

$$4) f'(x) = \left(\frac{\operatorname{tg} x}{\operatorname{ctg} x} \right)' = \frac{(\operatorname{tg} x)' \operatorname{ctg} x - \operatorname{tg} x \cdot (\operatorname{ctg} x)'}{\operatorname{ctg}^2 x} = \frac{\frac{\operatorname{ctg} x}{\cos^2 x} + \frac{\operatorname{tg} x}{\sin^2 x}}{\operatorname{ctg}^2 x} = \frac{\frac{1}{\sin x \cos x} + \frac{1}{\sin x \cos x}}{\operatorname{ctg}^2 x} = \frac{2}{\frac{\sin x \cos x}{\cos^2}} = \frac{2 \sin x}{\cos^3 x};$$

$$5) f'(x) = (\operatorname{tg} x \cdot \operatorname{ctg} x)' = \left(\frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x} \right)' = 1' = 0;$$

$$6) f'(x) = (\operatorname{tg} x + \operatorname{ctg} x)' = (\operatorname{tg} x)' + (\operatorname{ctg} x)' = \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} = \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} = -\frac{\cos^2 x - \sin^2 x}{\frac{1}{4} \sin^2 2x} = -\frac{4 \cos 2x}{\sin^2 2x}.$$