Zadatak 14. Riješi u skupu realnih brojeva jednadžbu $f'(x)=g'(x)$ ako je:

1) $f(x)=-\frac{1}{4}x^4+2x,\ g(x)=\frac{1}{3}x^3;$

2) $f(x)=\frac{1}{2}x^4,\ g(x)=-\frac{1}{2}x^2+3;$

3) $f(x)=\frac{1}{4}x^4+\frac{1}{3}x^3,\ g(x)=\frac{1}{2}x^2+10x.$

Rješenje. **1)** $f'(x)=\left(-\frac{1}{4}x^4+2x\right)'=\left(-\frac{1}{4}x^4\right)'+(2x)'=4\cdot\left(-\frac{1}{4}x^3\right)+2=-x^3+2,$

$g'(x)=\left(\frac{1}{3}x^3\right)'=3\cdot\frac{1}{3}x^2=x^2;$

$$-x^3+2=x^2$$
$$x^3+x^2-2=0$$
$$(x-1)\underbrace{(x^2+2x+2)}_{>0,\ \forall x\in\mathbf{R}}=0$$
$$x=1;$$

2) $f'(x)=\left(\frac{1}{2}x^4\right)'=4\cdot\frac{1}{2}x^3=2x^3,$

$g'(x)=\left(-\frac{1}{2}x^2+3\right)'=-\left(\frac{1}{2}x^2\right)'+3'=-2\cdot\frac{1}{2}x+0=-x;$

$$2x^3=-x$$
$$2x^3+x=0$$
$$x\underbrace{(2x^2+1)}_{>0,\ \forall x\in\mathbf{R}}=0$$
$$x=0;$$

3) $f'(x)=\left(\frac{1}{4}x^4+\frac{1}{3}x^3\right)'=\left(\frac{1}{4}x^4\right)+\left(\frac{1}{3}x^3\right)'=4\cdot\frac{1}{4}x^3+3\cdot\frac{1}{3}x^2=x^3+x^2,$

$g'(x)=\left(\frac{1}{2}x^2+10x\right)'=\left(\frac{1}{2}x^2\right)'+(10x)'=2\cdot\frac{1}{2}x+10=x+10;$

$$x^3+x^2=x+10$$
$$x^3+x^2-x-10=0$$
$$(x-2)\underbrace{(x^2+3x+5)}_{>0,\ \forall x\in\mathbf{R}}=0$$
$$x=2.$$