Zadatak 15. Riješi nejednadžbu $f'(x) > g'(x)$ ako je:

1) $f(x) = 2x^3 - x^2 + \sqrt{3}$,
$g(x) = x^3 - \frac{1}{2}x^2 - \sqrt{3}$;

2) $f(x) = \frac{2}{x}$, $g(x) = x - x^3$.

Rješenje. **1)** $f'(x) = (2x^3 - x^2 + \sqrt{3})' = (2x^3)' - (x^2)' + (\sqrt{3})' = 3 \cdot 2x^2 - 2x + 0 = 6x^2 - 2x$,

$g'(x) = \left(x^3 - \frac{1}{2}x^2 - \sqrt{3}\right)' = (x^3)' - \left(\frac{1}{2}x^2\right)' - (\sqrt{3})' = 3x^2 - 2 \cdot \frac{1}{2}x - 0 = 3x^2 - x$;

$$6x^2 - 2x > x^2 - x$$

$$3x^2 - x > 0$$

$$x(3x - 1) > 0$$

$$\begin{array}{cc} x < 0 & x > 0 \\ \underline{3x < 1} & \underline{3x > 1} \\ x \in \langle -\infty, 0\rangle & x \in \left\langle \frac{1}{3}, +\infty \right\rangle \end{array}$$

$$x \in \langle -\infty, 0\rangle \cup \left\langle \frac{1}{3}, +\infty \right\rangle;$$

2) $f'(x) = \left(\frac{2}{x}\right)' = (2x^{-1})' = -1 \cdot 2x^{-1-1} = -2x^{-2} = -\frac{2}{x^2}$,

$g'(x) = (x - x^3) = x' - (x^3)' = 1 - 3x^2$,

$$-\frac{2}{x^2} > 1 - 3x^2$$

$$-\frac{2}{x^2} - 1 + 3x^2 > 0$$

$$\frac{3x^4 - x^2 - 2}{x^2} > 0$$

$$3x^4 - x^2 - 2 > 0 \quad (\text{jer je } x^2 \geqslant 0\ \forall x \in \mathbf{R})$$

$$\underbrace{(3x^2 + 2)}_{>0,\ \forall x \in \mathbf{R}}(x^2 - 1) > 0$$

$$x^2 > 1$$

$$|x| > 1.$$