Zadatak 16. Odredi sve derivacije funkcija:

1) $f(x) = 2x^5 - 3x$; **2)** $f(x) = \dfrac{x^6}{90}$;

3) $f(x) = ax^4 + bx^3 + cx^2 + dx + e$.

Rješenje. **1)** $f(x) = 2x^5 - 3x$;

$$\begin{aligned}
f'(x) &= (2x^5 - 3x)' = (2x^5)' - (3x)' = 5 \cdot 2x^4 - 3 = 10x^4 - 3\\
f''(x) &= (10x^4 - 3)' = (10x^4)' - 3' = 4 \cdot 10x^3 = 40x^3\\
f'''(x) &= (40x^3)' = 3 \cdot 40x^2 = 120x^2\\
f^{iv}(x) &= (120x^2)' = 2 \cdot 120x = 240x\\
f^{(5)}(x) &= (240x)' = 240\\
f^{(n)}(x) &= (240)' = 0, \quad n \geqslant 6
\end{aligned}$$

2) $f(x) = \dfrac{x^6}{90}$;

$$\begin{aligned}
f'(x) &= \left(\frac{x^6}{90}\right)' = 6 \cdot \frac{x^5}{90} = \frac{x^5}{15}\\
f''(x) &= \left(\frac{x^5}{15}\right)' = 5 \cdot \frac{x^4}{15} = \frac{x^4}{3}\\
f'''(x) &= \left(\frac{x^4}{3}\right)' = 4 \cdot \frac{x^3}{3} = \frac{4x^3}{3}\\
f^{iv} &= \left(\frac{4x^3}{3}\right)' = 3 \cdot \frac{4x^2}{3} = 4x^2\\
f^{(5)}(x) &= (4x^2)' = 2 \cdot 4x = 8x\\
f^{(6)}(x) &= (8x)' = 8\\
f^{(n)}(x) &= 8' = 0, \quad n \geqslant 7
\end{aligned}$$

3) $f(x) = ax^4 + bx^3 + cx^2 + dx + e$;

$$\begin{aligned}
f'(x) &= (ax^4 + bx^3 + cx^2 + dx + e) = (ax^4)' + (bx^3)' + (cx^2)' + (dx)' + e'\\
&= 4 \cdot ax^3 + 3 \cdot bx^2 + 2 \cdot cx + d + 0 = 4ax^3 + 3bx^2 + 2cx + d\\
f''(x) &= (4ax^3 + 3bx^2 + 2cx + d)' = (4ax^3)' + (3bx^2)' + (2cx)' + d'\\
&= 3 \cdot 4ax^2 + 2 \cdot 3bx + 2c + 0 = 12ax^2 + 6bx + 2c\\
f'''(x) &= (12ax^2 + 6bx + 2c)' = (12ax^2) + (6bx)' + (2c)' = 2 \cdot 12ax + 6b + 0 = 24ax + 6b\\
f^{(iv)}(x) &= (24ax + 6b)' = 24a\\
f^{(n)}(x) &= (24a)' = 0, \quad n \geqslant 5
\end{aligned}$$