Zadatak 17. Odredi prve dvije derivacije funkcija:

1) $f(x) = \operatorname{tg} x + \operatorname{ctg} x$; **2)** $f(x) = \sin x - \operatorname{tg} x$;

3) $f(x) = \operatorname{tg} x \cdot \operatorname{ctg} x$.

Rješenje. **1)**

$$
\begin{aligned}
f'(x) &= (\operatorname{tg} x + \operatorname{ctg} x)' = (\operatorname{tg} x)' + (\operatorname{ctg} x)' = \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} \\
&= \frac{\sin^x - \cos^2 x}{\sin^2 x \cos^2 x} = \frac{-(\cos^2 x - \sin^2 x)}{\left(\frac{1}{2}\sin x \cos x\right)^{\frac{1}{2}}} = -\frac{\cos 2x}{\frac{1}{4}\sin^2 2x} \\
&= -4\frac{\cos 2x}{\sin^2 2x} = \frac{-4\operatorname{ctg} 2x}{\sin 2x} \\
f''(x) &= \left(-4\frac{\operatorname{ctg} 2x}{\sin 2x}\right)' \\
&= -4\frac{(\operatorname{ctg} 2x)' \sin 2x - \operatorname{ctg} 2x (\sin 2x)'}{\sin^2 2x} \\
&= -4\frac{\frac{-1}{\sin^2 2x} \cdot 2 \cdot \sin 2x - \frac{\cos 2x}{\sin 2x} \cdot \cos 2x \cdot 2}{\sin^2 2x} \\
&= -4\frac{\frac{-2}{\sin 2x} - \frac{2\cos^2 2x}{\sin 2x}}{\sin^2 2x} \\
&= \frac{8 + 8\cos^2 2x}{\sin^3 2x};
\end{aligned}
$$

2)

$$
\begin{aligned}
f'(x) &= (\sin x - \operatorname{tg} x)' = \cos x - \frac{1}{\cos^2 x} = \cos x - \frac{1}{\frac{\cos 2x + 1}{2}} = \cos x - \frac{2}{\cos 2x + 1} \\
f''(x) &= -\sin x - \frac{2'(\cos 2x + 1) - 2(\cos 2x + 1)'}{(\cos 2x + 1)^2} \\
&= -\sin x + \frac{-4\sin 2x}{4 \cdot \left(\frac{\cos 2x + 1}{2}\right)^2} \\
&= -\sin x - \frac{2\sin x \cos x}{(\cos^2 x\rangle^2} \\
&= -\sin x - \frac{2\sin x}{\cos^3 x};
\end{aligned}
$$

3) $f(x) = \operatorname{tg} x \cdot \operatorname{ctg} x = 1$, $f^{(n)}(x) = 0$, $\forall n \in \mathbf{N}$.