Zadatak 18. Odredi prve četiri derivacije funkcija:

1) $f(x)=\dfrac{x-1}{x+1}$; **2)** $f(x)=\dfrac{1}{1+x}$;

3) $f(x)=x\sin x$.

Rješenje. **1)**

$$f'(x)=\left(\frac{x-1}{x+1}\right)=\frac{x+1-(x-1)}{(x+1)^2}=\frac{x+1-x+1}{(x+1)^2}=\frac{2}{(x+1)^2}=2(x+1)^{-2}$$

$$f''(x)=-4(x+1)^{-3}=-\frac{4}{(x+1)^3}=-4(x+1)^{-3}$$

$$f'''(x)=12(x+1)^{-4}=\frac{12}{(x+1)^4}=12(x+1)^{-4}$$

$$f^{iv}(x)=-48(x+1)^{-5}=-\frac{48}{(x+1)^5}$$

$$f^{n}(x)=(-1)^{n+1}\frac{2\cdot n!}{(x+1)^{n+1}},\quad n\in\mathbf{N};$$

2)

$$f'(x)=\left(\frac{1}{1+x}\right)'=[(x+1)^{-1}]'=-(x+1)^{-2}=-\frac{1}{(x+1)^2}$$

$$f''(x)=2(x+1)^{-3}=\frac{2}{(x+1)^3}$$

$$f'''(x)=-6(x+1)^{-4}=-\frac{6}{(x+1)^4}$$

$$f^{iv}(x)=24(x+1)^{-5}=\frac{24}{(x+1)^5}$$

$$f^{n}(x)=(-1)^{n}\cdot\frac{n!}{(x+1)^{n+1}},\quad n\in\mathbf{N};$$

3)

$$f'(x)=(x\sin x)'=\sin x+x\cos x$$

$$f''(x)=\cos x+\cos x-x\sin x=2\cos x-x\sin x$$

$$f'''(x)=-2\sin x-\sin x-x\cos x=-3\sin x-x\cos x$$

$$f^{iv}(x)=-3\cos x-\cos x+x\sin x=-4\cos x+x\sin x.$$