Zadatak 2. Izračunaj derivacije sljedećih funkcija:

1) $f(x) = (x + \sqrt{x})^2$;
2) $f(x) = (x\sqrt{x} - 1)^3$;
3) $f(x) = (1 + \sqrt{x} + x)^2$;
4) $f(x) = (2x + 3)^2(3 - 2x)^3$;
5) $f(x) = (x - 1)^2(x + 1)^2$;
6) $f(x) = (1 + 2x)(x - 3)^2(2 - 5x)^3$;
7) $f(x) = (x - 1)^2(x + 1) - (x + 1)^2(x - 1)$;
8) $f(x) = (x^2 - 1)(x^4 + x^2 + 1) + (1 - x^2)^3$.

Rješenje. **1)** $f'(x) = [(x+\sqrt{x})^2]' = 2\cdot(x+\sqrt{x})\cdot(x+\sqrt{x})' = 2(x+\sqrt{x})\left(1+\frac{1}{2\sqrt{x}}\right) = 2\sqrt{x}(1+\sqrt{x})\frac{2\sqrt{x}+1}{2\sqrt{x}} = (1+\sqrt{x})(1+2\sqrt{x}) = 2x+3\sqrt{x}+1$;

2) $f'(x) = [(x\sqrt{x}-1)^3]' = 3\cdot(x\sqrt{x}-1)^{3-1}\cdot(x\sqrt{x}-1)' = 3(x\sqrt{x}-1)^2\left(\frac{3}{2}\sqrt{x}\right) = \frac{9}{2}\sqrt{x}(x\sqrt{x}-1)^2$;

3) $f'(x) = [(1+\sqrt{x}+x)^2]' = 2\cdot(1+\sqrt{x}+x)\cdot(1+\sqrt{x}+x) = 2(1+\sqrt{x}+x)\left(\frac{1}{2\sqrt{x}}+1\right)$;

4) $f'(x) = [(2x+3)^2(3-2x)^3]' = 2\cdot(2x+3)^{2-1}(2x+3)'(3-2x)^3 + (2x+3)^2\cdot 3\cdot(3-2x)^{3-1}\cdot(3-2x)' = 2(2x+3)\cdot 2(3-2x)^3 - 6(2x+3)^2(3-2x)^2 = (2x+3)(3-2x)^2[4(3-2x)-6(2x+3)] = (2x+3)(3-2x)^2[12-8x-12x-18] = -(2x+3)(3-2x)^2\cdot 2(10x+3) = -2(2x+3)(3-2x)^2(10x+3)$;

5) $f'(x) = [(x-1)^2(x+1)^2]' = 2(x-1)^{2-1}\cdot(x-1)'(x+1)^2 + (x-1)^2\cdot 2\cdot(x+1)^{2-1}\cdot(x+1)' = 2(x-1)(x+1)^2 + 2(x-1)^2(x+1) = 2(x-1)(x+1)[x+1+x-1] = 2(x^2-1)2x = 4x(x^2-1)$;

6) $f'(x) = [(1+2x)(x-3)^2(2-5x)^3]' = (1+2x)'(x-3)^2(2-5x)^3 + (1+2x)\cdot 2\cdot(x-3)^{2-1}(x-3)'(2-5x)^3 + (1+2x)(x-3)^2\cdot 3\cdot(2-5x)^{3-1}(2-5x)' = 2(x-3)^2(2-5x)^3 + 2(1+2x)(x-3)(2-5x)^3 - 15(1+2x)(x-3)^2(2-5x)^2 = (x-3)(2-5x)^2[2(x-3)(2-5x)+2(1+2x)(2-5x)-15(1+2x)(x-3)] = (x-3)(2-5x)^2[4x-10x^2-12+30x+4-10x+8x-20x^2-45x+45-30x^2+45x] = (x-3)(2-5x)^2(-60x^2+107x+37)$;

7) $f'(x) = [(x-1)^2(x+1)-(x+1)^2(x-1)]' = [(x^2-1)(x-1-x-1)]' = [-2(x^2-1)]' = -2(x^2-1)' = -2(2x) = -4x$;

8) $f'(x) = [(x^2-1)(x^4+x^2+1)+(1-x^2)^3]' = [x^6-1+(1-x^2)^3]' = (x^6)'-1'+[(1-x^2)^3]' = 6x^5-0+3\cdot(1-x^2)^{3-1}\cdot(1-x^2)' = 6x^5+3(1-x^2)^2\cdot(-2x) = 6x^5-6x(1-x^2)^2 = 6x[x^4-(1-x^2)^2] = 6x(x^2-1+x^2)(x^2+1-x^2) = 6x(2x^2-1)$.