

Zadatak 3. Deriviraj sljedeće funkcije:

$$1) f(x) = \sqrt{x^2 + 1};$$

$$2) f(x) = \sqrt{1 - 2x};$$

$$3) f(x) = \sqrt{1 + 3x^2};$$

$$4) f(x) = \sqrt{4 - 3x^2};$$

$$5) f(x) = \sqrt{x + \sqrt{x}};$$

$$6) f(x) = \sqrt{1 - \sqrt{x}};$$

$$7) f(x) = \sqrt[4]{(x^2 + 1)^3};$$

$$8) f(x) = \sqrt[3]{1 - 2x}.$$

Rješenje.

$$1) f'(x) = [\sqrt{x^2 + 1}]' = [(x^2 + 1)^{\frac{1}{2}}]' = \frac{1}{2}(x^2 + 1)^{\frac{1}{2}-1}(x^2 + 1)' = \frac{1}{2}(x^2 + 1)^{-\frac{1}{2}} \cdot 2x = \frac{x}{\sqrt{x^2 + 1}};$$

$$2) f'(x) = [\sqrt{1 - 2x}]' = [(1 - 2x)^{\frac{1}{2}}]' = \frac{1}{2}(1 - 2x)^{\frac{1}{2}-1}(1 - 2x)' = \frac{1}{2}(1 - 2x)^{-\frac{1}{2}}(-2) = -\frac{1}{\sqrt{1 - 2x}};$$

$$3) f'(x) = [\sqrt{1 + 3x^2}]' = [(3x^2 + 1)^{\frac{1}{2}}]' = \frac{1}{2}(3x^2 + 1)^{\frac{1}{2}-1}(3x^2 + 1)' = \frac{1}{2}(3x^2 + 1)^{-\frac{1}{2}}6x = \frac{3x}{\sqrt{3x^2 + 1}};$$

$$4) f'(x) = [\sqrt{4 - 3x^2}]' = [(4 - 3x^2)^{\frac{1}{2}}]' = \frac{1}{2}(4 - 3x^2)^{\frac{1}{2}-1}(4 - 3x^2)' = \frac{1}{2}(4 - 3x^2)^{-\frac{1}{2}} \cdot (-6x) = -\frac{3x}{\sqrt{4 - 3x^2}};$$

$$5) f'(x) = [\sqrt{x + \sqrt{x}}]' = [(x + x^{\frac{1}{2}})^{\frac{1}{2}}]' = \frac{1}{2}(x + x^{\frac{1}{2}})^{\frac{1}{2}-1}(x + x^{\frac{1}{2}})' = \frac{1}{2}(x + x^{\frac{1}{2}})^{-\frac{1}{2}} \cdot \left(1 + \frac{1}{2}x^{-\frac{1}{2}}\right) = \frac{1}{2\sqrt{x + \sqrt{x}}} \cdot \left(1 + \frac{1}{2\sqrt{x}}\right) = \frac{1 + 2\sqrt{x}}{4\sqrt{x^2 + x\sqrt{x}}};$$

$$6) f'(x) = [\sqrt{1 - \sqrt{x}}]' = [(1 - x^{\frac{1}{2}})^{\frac{1}{2}}]' = \frac{1}{2}(1 - x^{\frac{1}{2}})^{\frac{1}{2}-1}(1 - x^{\frac{1}{2}})' = \frac{1}{2}(1 - x^{\frac{1}{2}})^{-\frac{1}{2}} \left(-\frac{1}{2}\right)x^{-\frac{1}{2}} = \frac{1}{2\sqrt{1 - \sqrt{x}}} \left(-\frac{1}{2}\right) \frac{1}{\sqrt{x}} = -\frac{1}{4\sqrt{x - x\sqrt{x}}};$$

$$7) f'(x) = [\sqrt[4]{(x^2 + 1)^3}]' = [(x^2 + 1)^{\frac{3}{4}}]' = \frac{3}{4}(x^2 + 1)^{\frac{3}{4}-1}(x^2 + 1)' = \frac{3}{4}(x^2 + 1)^{-\frac{1}{4}}2x = \frac{3x}{2\sqrt[4]{x^2 + 1}};$$

$$8) f'(x) = [\sqrt[3]{1 - 2x}]' = [(1 - 2x)^{\frac{1}{3}}]' = \frac{1}{3}(1 - 2x)^{\frac{1}{3}-1}(1 - 2x)' = \frac{1}{3}(1 - 2x)^{-\frac{2}{3}} \cdot (-2) = -\frac{2}{3\sqrt[3]{(1 - 2x)^2}}.$$