Zadatak 4. Deriviraj funkcije:

1) $f(x) = \dfrac{x-2}{\sqrt{x^2-1}}$;

2) $f(x) = \dfrac{\sqrt{x+1}}{\sqrt{x-1}}$;

3) $f(x) = \sqrt{\dfrac{x^2-1}{x^2+1}}$;

4) $f(x) = \sqrt{\dfrac{3}{2x^2-1}}$;

5) $f(x) = (x-1)\sqrt{x^2+1}$;

6) $f(x) = (x+1)^2\sqrt{x-1}$;

7) $f(x) = \dfrac{x-2}{2}\sqrt{\dfrac{8}{2x-4}}$;

8) $f(x) = (x-1)\sqrt{\dfrac{1}{x^2-1}}$;

9) $f(x) = x \cdot \dfrac{\sqrt{x-2\sqrt{x-1}}}{\sqrt{x-1}-1}$;

10) $f(x) = \dfrac{\sqrt{1+\left(\dfrac{x^2-1}{2x}\right)^2}}{(x^2+1)\cdot\dfrac{1}{x^2}}$.

Rješenje. **1)** $f'(x) = \left[\dfrac{x-2}{\sqrt{x^2-1}}\right]' = [(x-2)(x^2-1)^{-\frac{1}{2}}]' = (x-2)'(x^2-1)^{-\frac{1}{2}} + (x-2)[(x^2-1)^{-\frac{1}{2}}]' = (x^2-1)^{-\frac{1}{2}} + (x-2)\left(-\dfrac{1}{2}\right)(x^2-1)^{-\frac{1}{2}-1}(x^2-1)' = \dfrac{1}{\sqrt{x^2-1}} - \dfrac{1}{2}(x-2)(x^2-1)^{-\frac{3}{2}}\cdot 2x = \dfrac{1}{\sqrt{x^2-1}} - \dfrac{x(x-2)}{(x^2-1)^{\frac{3}{2}}} = \dfrac{1}{\sqrt{x^2-1}} - \dfrac{(x-2)x}{(x^2-1)\sqrt{x^2-1}} = \dfrac{x^2-1-x^2+2x}{\sqrt{(x^2-1)^3}} = \dfrac{2x-1}{\sqrt{(x^2-1)^3}}$;

2) $f'(x) = \left[\dfrac{\sqrt{x+1}}{\sqrt{x-1}}\right]' = [(x+1)^{\frac{1}{2}}(x-1)^{-\frac{1}{2}}]' = [(x+1)^{\frac{1}{2}}]'(x-1)^{-\frac{1}{2}} + (x+1)^{\frac{1}{2}}[(x-1)^{-\frac{1}{2}}]' = \dfrac{1}{2}(x+1)^{-\frac{1}{2}}(x-1)^{-\frac{1}{2}} + (x+1)^{\frac{1}{2}}\cdot\left(-\dfrac{1}{2}\right)(x-1)^{-\frac{3}{2}} = \dfrac{1}{2(x+1)^{\frac{1}{2}}(x-1)^{\frac{1}{2}}} - \dfrac{\sqrt{x+1}}{2(x-1)^3 2} = \dfrac{1}{2\sqrt{(x+1)(x-1)}} - \dfrac{\sqrt{x+1}}{2(x-1)\sqrt{x-1}} = \dfrac{x-1-\sqrt{x+1}\sqrt{x+1}}{2(x-1)\sqrt{(x+1)(x-1)}} = \dfrac{x-1-x-1}{2(x-1)\sqrt{x^2-1}} = -\dfrac{1}{(x-1)\sqrt{x^2-1}} = -\dfrac{1}{(x-1)^2}\sqrt{\dfrac{x-1}{x+1}}$;

3) $f'(x) = \left[\sqrt{\dfrac{x^2-1}{x^2+1}}\right]' = [(x^2-1)^{\frac{1}{2}}(x^2+1)^{-\frac{1}{2}}]' = [(x^2-1)^{\frac{1}{2}}]'(x^2+$

$1)^{-\frac{1}{2}} + (x^2-1)^{\frac{1}{2}}[(x^2+1)^{-\frac{1}{2}}]' = \frac{1}{2}\cdot(x^2-1)^{\frac{1}{2}-1}\cdot(x^2-1)'(x^2+1)^{-\frac{1}{2}} - \frac{1}{2}(x^2-1)^{\frac{1}{2}}(x^2+1)^{-\frac{1}{2}-1}\cdot(x^2+1)' = x(x^2-1)^{-\frac{1}{2}}(x^2+1)^{-\frac{1}{2}} - x(x^2-1)^{\frac{1}{2}}(x^2+1)^{-\frac{3}{2}} = \frac{x}{(x^2-1)^{\frac{1}{2}}(x^2+1)^{\frac{1}{2}}} - \frac{x\sqrt{x^2-1}}{(x^2+1)^{\frac{3}{2}}} = \frac{x}{\sqrt{(x^2-1)(x^2+1)}} - \frac{x\sqrt{x^2-1}}{(x^2+1)\sqrt{x^2+1}} = \frac{x(x^2+1)-x\sqrt{(x^2-1)(x^2-1)}}{(x^2+1)\sqrt{(x^2-1)(x^2+1)}} = \frac{x^3+x-x(x^2-1)}{(x^2+1)\sqrt{x^4-1}} = \frac{x^3+x-x^3+x}{(x^2+1)\sqrt{x^4-1}} = \frac{2x}{(x^2+1)\sqrt{x^2-1}\sqrt{x^2-1}} = \frac{2x}{(x^2+1)^2}\sqrt{\frac{x^2+1}{x^2-1}};$

4) $f'(x) = \left(\sqrt{\frac{3}{2x^2-1}}\right)' = (\sqrt{3}(2x^2-1)^{-\frac{1}{2}})' = \sqrt{3}\left(-\frac{1}{2}\right)(2x^2-1)^{-\frac{1}{2}-1}\cdot(2x^2-1)' = -\frac{\sqrt{3}}{2}(2x^2-1)^{-\frac{3}{2}}\cdot 4x = -\frac{2x\sqrt{3}}{(2x^2-1)^{\frac{3}{2}}} = -\frac{2x\sqrt{3}}{\sqrt{(2x^2-1)^3}};$

5) $f'(x) = [(x-1)\sqrt{x^2+1}]'[(x-1)(x^2+1)^{\frac{1}{2}}]' = (x-1)'(x^2+1)^{\frac{1}{2}} + (x-1)[(x^2+1)^{\frac{1}{2}}]' = (x^2+1)^{\frac{1}{2}} + (x-1)\cdot\frac{1}{2}(x^2+1)^{\frac{1}{2}-1}(x^2+1)' = (x^2+1)^{\frac{1}{2}} + \frac{1}{2}(x-1)(x^2+1)^{-\frac{1}{2}}\cdot 2x = (x^2+1)^{\frac{1}{2}} + \frac{x(x-1)}{(x^2+1)^{\frac{1}{2}}} = \frac{x^2+1+x^2-x}{\sqrt{x^2+1}} = \frac{2x^2-x+1}{\sqrt{x^2+1}};$

6) $f'(x) = [(x+1)^2\sqrt{x-1}]' = [(x+1)^2(x-1)^{\frac{1}{2}}]' = [(x+1)^2]'(x-1)^{\frac{1}{2}} + (x+1)^2[(x-1)^{\frac{1}{2}}]' = 2\cdot(x+1)\cdot(x+1)'(x-1)^{\frac{1}{2}} + (x+1)^2\cdot\frac{1}{2}\cdot(x-1)^{\frac{1}{2}-1}\cdot(x-1)' = 2(x+1)(x-1)^{\frac{1}{2}} + \frac{1}{2}(x+1)^2(x-1)^{-\frac{1}{2}} = 2(x+1)\sqrt{x-1} + \frac{(x+1)^2}{2(x-1)^{\frac{1}{2}}} = 2(x+1)\sqrt{x-1} + \frac{(x+1)^2}{2\sqrt{x-1}} = \frac{4(x+1)(x-1)+x^2+2x+1}{2\sqrt{x-1}} = \frac{4x^2-4+x^2+2x+1}{2\sqrt{x-1}} = \frac{5x^2+2x-3}{2\sqrt{x-1}};$

7) $f'(x) = \left[\frac{x-2}{2}\sqrt{\frac{8}{2x-4}}\right]' = \left[\frac{x-2}{2}\cdot\frac{2\sqrt{2}}{\sqrt{2}\sqrt{x-2}}\right]' = (\sqrt{x-2})' = [(x-2)^{\frac{1}{2}}]' = \frac{1}{2}(x-2)^{\frac{1}{2}-1} = \frac{1}{2}(x-2)^{-\frac{1}{2}} = \frac{1}{2(x-2)^{\frac{1}{2}}} = \frac{1}{2\sqrt{x-2}};$

8) $f'(x) = \left[(x-1)\sqrt{\frac{1}{x^2-1}}\right]' = \left[\frac{x-1}{\sqrt{x-1}\sqrt{x+1}}\sqrt{\frac{x-1}{x+1}}\right]' = [(x-1)^{\frac{1}{2}}(x+1)^{-\frac{1}{2}}]' = [(x-1)^{\frac{1}{2}}]'(x+1)^{-\frac{1}{2}} + (x-1)^{\frac{1}{2}}[(x+1)^{-\frac{1}{2}}]' = \frac{1}{2}(x-1)^{\frac{1}{2}-1}(x+1)^{-\frac{1}{2}} + (x-1)^{\frac{1}{2}}\cdot\left(-\frac{1}{2}\right)(x+1)^{-\frac{1}{2}-1} = \frac{1}{2}(x-1)^{-\frac{1}{2}}(x+1)^{-\frac{1}{2}} - \frac{1}{2}(x-1)^{\frac{1}{2}}(x+1)^{-\frac{3}{2}} = \frac{1}{2\sqrt{x-1}\sqrt{x+1}} - \frac{\sqrt{x-1}}{2\sqrt{(x+1)^3}}\,\frac{x+1-\sqrt{x-1}\sqrt{x-1}}{2\sqrt{x-1}\sqrt{(x+1)^3}} = \frac{x+1-x+1}{2\sqrt{x-1}\sqrt{(x+1)^3}} = \frac{1}{(x+1)\sqrt{x-1}\sqrt{x+1}} = \frac{1}{(x+1)^2}\sqrt{\frac{x+1}{x-1}};$

9) $f(x) = x \cdot \dfrac{\sqrt{x-2\sqrt{x-1}}}{\sqrt{x-1}-1} = x\dfrac{\sqrt{x-2\sqrt{x+1}}}{\sqrt{(\sqrt{x-1}-1)^2}} = x\sqrt{\dfrac{x-2\sqrt{x-1}}{(\sqrt{x-1}-1)^2}} =$

$x\sqrt{\dfrac{x-2\sqrt{x-1}}{x-1-2\sqrt{x-1}+1}} = x\sqrt{\dfrac{x-2\sqrt{x-1}}{x-2\sqrt{x-1}}} = \begin{cases} x, & x > 2 \\ -x, & x \in [1,2\rangle \end{cases}$;

$f'(x) = \begin{cases} 1, & x > 2 \\ -1, & 1 \leqslant x < 2 \end{cases}$

$x - 1 \geqslant 0 \implies x \geqslant 1, \quad \sqrt{x-1} - 1 \neq 0 \implies \sqrt{x-1} \neq 1$

$\implies x - 1 \neq 1 \implies x \neq 2,$

$x - 2\sqrt{x-1} \geqslant 0, \quad x \geqslant 2\sqrt{x-1},$

$x^2 \geqslant 4x - 4, \quad x^2 - 4x + 4 \geqslant 0,$

$(x-2)^2 \geqslant 0$, $x \in \mathbf{R}$, $D(f) = [1, +\infty\rangle \setminus \{2\}$;

10) $f'(x) = \left(\dfrac{\sqrt{1+\left(\dfrac{x^2-1}{2x}\right)^2}}{(x^2+1)\cdot\dfrac{1}{x^2}}\right)' = \left(\dfrac{\sqrt{\dfrac{4x^2+x^4-2x^2+1}{4x^2}}}{\dfrac{x^2+1}{x^2}}\right)'$

$= \left(\dfrac{\sqrt{\dfrac{x^4+2x^2+1}{4x^2}}}{\dfrac{x^2+1}{x^2}}\right)' = \left(\dfrac{\pm\dfrac{x^2+1}{2x}}{\dfrac{x^2+1}{x^2}}\right)' = \left(\pm\dfrac{x}{2}\right)' = \pm\dfrac{1}{2}.$