Zadatak 7. Deriviraj funkcije:

1) $f(x) = \sin^3 2x \cdot \cos^2 3x$;

2) $f(x) = 4\sin^2 x - 8\sin x + 3$;

3) $f(x) = \sin(x^2 + 1)$;

4) $f(x) = 2\sin x + 3\sin^3 x$;

5) $f(x) = \sin^2(x^3)$;

6) $f(x) = 2x\,\mathrm{tg}\,2x$;

7) $f(x) = \sin\sqrt{1+x^2}$;

8) $f(x) = \mathrm{tg}\,x^2 \cdot \mathrm{ctg}\,x^2$.

Rješenje. **1)** $'f(x) = (\sin^3 2x \cdot \cos^2 3x)' = (\sin^3 2x)' \cdot cos^2 3x + \sin^3 2x \cdot (\cos^2 3x)' ==$
$3\sin^2 2x \cos 2x \cdot 2\cos^2 3x + \sin^3 2x 2\cos 3x(-\sin 3x)\cdot 3 = 6\sin^2 2x\cos 3x(\cos 2x\cos 3x - \sin 2x\sin 3x) = 6\sin^2 2x\cos 3x\cos 5x$;

2) $f'(x) = (4\sin^2 x - 8\sin x + 3)' = 8\sin x\cos x - 8\cos x = 8\cos x(\sin x - 1)$;

3) $'f(x) = [\sin(x^2+1)]' = 2x\cos(x^2+1)$;

4) $f'(x) = (2\sin x + 3\sin^3 x)' = 2\cos x + 9\sin^2 x\cos x = \cos x(2 + 9\sin^2 x)$;

5) $f'(x) = [\sin^2(x^3)]' = 2\sin(x^3)\cos(x^3)\cdot 3x^2 = 3x^2\sin(2x^3)$;

6) $f'(x) = (2x\,\mathrm{tg}\,2x)' = 2\,\mathrm{tg}\,2x + 2x\cdot\dfrac{1}{\cos^2 2x}\cdot 2 = 2\,\mathrm{tg}\,2x + \dfrac{4x}{\cos^2 2x}$;

7) $f'(x) = (\sin\sqrt{1+x^2})' = \cos\sqrt{1+x^2}\cdot\dfrac{1}{2\sqrt{1+x^2}}\cdot 2x = \dfrac{x}{\sqrt{1+x^2}}\cos\sqrt{1+x^2}$;

8) $f'(x) = (\mathrm{tg}\,x^2\cdot\mathrm{ctg}\,x^2)' = (\mathrm{tg}\,x^2)'\,\mathrm{ctg}\,x^2 + \mathrm{tg}\,x^2\cdot(\mathrm{ctg}\,x^2)' = \dfrac{1}{\cos^2 x^2}\cdot$
$2x\cdot\mathrm{ctg}\,x^2 + \mathrm{tg}\,x^2\left(-\dfrac{1}{\sin^2 x^2}\right)\cdot 2x = \dfrac{2x}{\sin x^2\cos x^2} - \dfrac{2x}{\sin x^2\cos x^2} = 0$.