

**Zadatak 8.** Deriviraj funkcije:

1)  $f(x) = \cos^4 3x - \sin^4 3x;$

2)  $f(x) = \sin 4x \cos 4x;$

3)  $f(x) = \operatorname{tg} 2x - \operatorname{ctg} 2x;$

4)  $f(x) = 3 \sin x \cos^2 x + \sin^3 x;$

5)  $f(x) = \sin^6 x + \cos^6 x;$

6)  $f(x) = (\sin x \sin 2x - \cos x \cos 2x)^3;$

7)  $f(x) = (\cos 3x \cos 2x - \sin 3x \sin 2x)^2;$

8)  $f(x) = 3 \sin\left(\frac{\pi}{3} - x\right) \cos\left(\frac{\pi}{3} + x\right);$

9)  $f(x) = -\frac{1}{2} \cos\left(\frac{\pi}{4} - x\right) \cos\left(\frac{\pi}{4} + x\right).$

**Rješenje.**

1)  $f(x) = (\cos^4 3x - \sin^4 3x)' = [(\cos^2 3x - \sin^2 3x)(\cos^2 3x + \sin^2 3x)]' =$   
 $(\cos 6x)' = -6 \sin 6x;$

2)  $f'(x) = (\sin 4x \cos 4x)' = \left(\frac{1}{2} \sin 8x\right)' = \frac{1}{2} \cdot 8 \cos 8x = 4 \cos 8x;$

3)  $f'(x) = (\operatorname{tg} 2x - \operatorname{ctg} 2x)' = \left[\frac{\sin 2x}{\cos 2x} - \frac{\cos 2x}{\sin 2x}\right]' = \left[\frac{\sin^2 2x - \cos^2 2x}{\sin 2x \cos 2x}\right]' =$   
 $\left(\frac{-2 \cos 4x}{\sin 4x}\right)' = (-2 \operatorname{ctg} 4x)' = -2 \cdot \frac{-1}{\sin^2 4x} \cdot 4 = \frac{8}{\sin^2 4x};$

4)  $f'(x) = (3 \sin x \cos^2 x + \sin^3 x)' = 3(\cos^3 x + \sin x \cdot 2 \cos x(-\sin x)) +$   
 $3 \sin^2 x \cos x = 3 \cos^3 x - 6 \sin^2 x \cos x + 3 \sin^2 x \cos x = 3 \cos^3 x - 3 \sin^2 x \cos x =$   
 $3 \cos x(\cos^2 x - \sin^2 x) = 3 \cos x \cos 2x;$

5)  $f'(x) = (\sin^6 x + \cos^6 x)' = [(\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cos^2 x +$   
 $\cos^4 x)]' = (\sin^4 x + 2 \sin^2 x \cos^2 x + \cos^4 x - 3 \sin^2 x \cos^2 x)' =$   
 $= \left[(\sin^2 x + \cos^2 x)^2 - \frac{3}{4} \sin^2 2x\right]' = \left(1 - \frac{3}{4} \sin^2 2x\right)' = -\frac{3}{4} \cdot 2 \sin 2x \cos 2x.$

$2 = -\frac{3}{2} \sin 4x;$

6)  $f'(x) = [(\sin x \sin 2x - \cos x \cos 2x)^3]' = (-\cos^3 3x)' = -3 \cos^2 3x(-\sin 3x) \cdot$   
 $3 = 9 \sin 3x \cos^2 3x;$

7)  $f'(x) = [(\cos 3x \cos 2x - \sin 3x \sin 2x)^2]' = (\cos^2 5x)' = 2 \cos 5x(-\sin 5x) =$   
 $-5 \sin 10x;$

8)  $f'(x) = \left(3 \sin\left(\frac{\pi}{3} - x\right) \cos\left(\frac{\pi}{3} + x\right)\right)' =$   
 $= \left[3 \left(\sin \frac{\pi}{3} \cos x - \cos \frac{\pi}{3} \sin x\right) \left(\cos \frac{\pi}{3} \cos x - \sin \frac{\pi}{3} \sin x\right)\right]' =$   
 $= \left[3 \left(\frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x\right) \left(\frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x\right)\right]' =$   
 $= \left[\frac{3}{4}(\sqrt{3} \cos x - \sin x)(\cos x - \sqrt{3} \sin x)\right]' =$   
 $= \left[\frac{3}{4}(\sqrt{3} \cos^2 x - \sin x \cos x - 3 \sin x \cos x + \sqrt{3} \sin^2 x)\right]' = \left[\frac{3}{4}(\sqrt{3} - 2 \sin 2x)\right]'$

$$\frac{3}{4}(-2 \cos 2x) \cdot 2 = -3 \cos 2x;$$

$$\begin{aligned}9) f'(x) &= \left[ -\frac{1}{2} \cos \left( \frac{\pi}{4} - x \right) \cos \left( \frac{\pi}{4} + x \right) \right]' \\&= \left[ -\frac{1}{2} \left( \cos \frac{\pi}{4} \cos x + \sin \frac{\pi}{4} \sin x \right) \left( \cos \frac{\pi}{4} \cos x - \sin \frac{\pi}{4} \sin x \right) \right]' \\&= \left[ -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} (\cos x + \sin x)(\cos x - \sin x) \right]' = \left[ -\frac{1}{4}(\cos^2 x - \sin^2 x) \right]' = \\&\left( -\frac{1}{4} \cos 2x \right)' = -\frac{1}{4}(-\sin 2x) \cdot 2 = \frac{1}{2} \sin 2x.\end{aligned}$$