Zadatak 10. Deriviraj funkcije:

1) $f(x) = \dfrac{\sin^4 x - \cos^4 x}{\sin^2 x}$;

2) $f(x) = \dfrac{\operatorname{ctg}\frac{x}{2} - \operatorname{tg}\frac{x}{2}}{\left(\operatorname{ctg}\frac{x}{2} - \operatorname{tg}\frac{x}{2}\right)^2 + 4}$;

3) $f(x) = \dfrac{4}{3}\operatorname{ctg} x - \dfrac{\cos x}{3\sin^3 x}$;

4) $f(x) = \dfrac{1 - \sin x}{\cos 2x}$;

5) $f(x) = \dfrac{\sin 2x}{1 + \cos x}$;

6) $f(x) = 3\cos(x^2 - \pi)$;

7) $f(x) = \dfrac{\sin x}{\cos^2 x}$;

8) $f(x) = \dfrac{1 + \cos 2x}{1 - \cos 2x}$;

9) $f(x) = \dfrac{2\sin x + \sin 2x}{2\sin x - \sin 2x}$;

10) $f(x) = \dfrac{\sin 3x + \sin 5x}{\cos 3x + \cos 5x}$.

Rješenje. **1)** $f'(x) = \left(\dfrac{\sin^4 x - \cos^4 x}{\sin^2 x}\right)' = \left[\dfrac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)}{\sin^2 x}\right]' = \left(\dfrac{\sin^2 x - \cos^2 x}{\sin^2 x}\right)' = (1 - \operatorname{ctg}^2 x)' = -2\operatorname{ctg} x \cdot \left(-\dfrac{1}{\sin^2 x}\right) = \dfrac{2\cos x}{\sin^3 x}$;

2) $f'(x) = \left(\dfrac{\operatorname{ctg}\frac{x}{2} - \operatorname{tg}\frac{x}{2}}{\left(\operatorname{ctg}\frac{x}{2} - \operatorname{tg}\frac{x}{2}\right)^2 + 4}\right)' = \left(\dfrac{\frac{\cos x}{\sin\frac{x}{2}\cos\frac{x}{2}}}{\frac{\cos^2 x}{\sin^2\frac{x}{2}\cos^2\frac{x}{2}} + 4}\right)'$

$= \left(\dfrac{\frac{2\cos x}{\sin x}}{\frac{4\cos^2 x}{\sin^2 x} + 4}\right)' = \left(\dfrac{\frac{2\cos x}{\sin x}}{\frac{4(\cos^2 x + \sin^2 x)}{\sin^2 x}}\right)' = \left(\dfrac{1}{4}\sin 2x\right)' = \dfrac{1}{4}\cos 2x \cdot 2 = \dfrac{1}{2}\cos 2x$;

3) $f'(x) = \left(\dfrac{4}{3}\operatorname{ctg} x - \dfrac{\cos x}{3\sin^3 x}\right)' = -\dfrac{4}{3\sin^2 x} - \dfrac{1}{3}\dfrac{(\cos x)'\sin^3 x - \cos x(\sin^3 x)'}{(\sin^3 x)^2} = -\dfrac{4}{3\sin^2 x} - \dfrac{1}{3}\cdot\dfrac{-\sin^4 x - \cos^2 x \cdot 3\sin^2 x}{\sin^6 x} = -\dfrac{4}{3\sin^2 x} + \dfrac{\sin^4 x + 3\sin^2 x\cos^2 x}{3\sin^6 x} = \dfrac{-4\sin^4 x + \sin^4 x + 3\sin^2 x\cos^2}{3\sin^6 x} = \dfrac{3\sin^2 x\cos^2 x - 3\sin^4 x}{3\sin^6 x}$

$= \dfrac{3\sin^2 x(\cos^2 x - \sin^2 x)}{3\sin^6 x} = \dfrac{\cos 2x}{\sin^4 x}$;

4) $f'(x) = \left(\frac{1-\sin x}{\cos 2x}\right)' = \frac{-\cos x \cos 2x - (1-\sin x)(-\sin 2x)\cdot 2}{\cos^2 2x}$
$= \frac{-\cos x \cos 2x - 2\sin x \sin 2x + 2\sin 2x}{\cos^2 2x}$;

5) $f'(x) = \left(\frac{\sin 2x}{1+\cos x}\right)' = \frac{2\cos 2x(1+\cos x) + \sin 2x \sin x}{(1+\cos x)^2}$
$= \frac{2\cos 2x + 2\cos x \cos 2x + \sin x \sin 2x}{(1+\cos x)^2}$;

6) $f'(x) = [3\cos(x^2-\pi)]' = [3\cos(\pi - x^2)]' = [-3\cos x^2]' = 3\sin x^2 \cdot 2x = 6x\sin x^2$;

7) $f'(x) = \left(\frac{\sin x}{\cos^2 x}\right)' = \frac{\cos^3 x + \sin^2 x \cdot 2\cos x}{\cos^4 x} = \frac{\cos^2 x + 2\sin^2 x}{\cos^3 x} = \frac{1+\cos^2 x}{\cos^3 x}$;

8) $f'(x) = \left(\frac{1+\cos 2x}{1-\cos 2x}\right)' = \left(\frac{2\cos^2 x}{2\sin^2 x}\right)' = (\mathrm{ctg}^2 x)' = 2\,\mathrm{ctg}\, x\left(-\frac{1}{\sin^2 x}\right) = -\frac{2\cos x}{\sin^3 x}$;

9) $f'(x) = \left(\frac{2\sin x + \sin 2x}{2\sin x - \sin 2x}\right)' = \left(\frac{2\sin x + 2\sin x \cos x}{2\sin x - 2\sin x \cos x}\right)' = \left(\frac{1+\cos x}{1-\cos x}\right)' = \left(\frac{2\cos^2 \frac{x}{2}}{2\sin^2 \frac{x}{2}}\right)' = \left(\mathrm{ctg}^2 \frac{x}{2}\right)' = 2\,\mathrm{ctg}\,\frac{x}{2}\left(-\frac{1}{\sin^2 \frac{x}{2}}\right)\cdot\frac{1}{2} = -\frac{\cos\frac{x}{2}}{\sin^3\frac{x}{2}}$;

10) $f'(x) = \left(\frac{\sin 3x + \sin 5x}{\cos 3x + \cos 5x}\right)' = \left(\frac{2\sin 4x \cos x}{2\cos 4x \cos x}\right)' = (\mathrm{tg}\, 4x)' = \frac{4}{\cos^2 4x}$.