**Zadatak 11.** Deriviraj sljedeće funkcije:

**1)** $f(x)=\dfrac{(x-2)^2+8x}{4-x^2}$;

**2)** $f(x)=\dfrac{(3x^2-1)^2-(x^2+1)^2}{4x^2-4x^4}$;

**3)** $f(x)=\dfrac{(18x^5-2x)^2}{9x^4+6x^2+1}$;

**4)** $f(x)=\dfrac{1}{\sqrt[3]{x+\sqrt{x}}}$;

**5)** $f(x)=\sqrt[3]{1+x\sqrt{x+3}}$;

**6)** $f(x)=x\sqrt{\dfrac{1-x}{1+x^2}}$.

***Rješenje.*** **1)** $f'(x)=\left(\dfrac{(x-2)^2+8x}{4-x^2}\right)'=\left(\dfrac{x^2-4x+4+8x}{(2-x)(2+x)}\right)'=\left(\dfrac{x^2+4x+4}{(2-x)(2+x)}\right)'=\left(\dfrac{(x+2)^2}{(2-x)(2+x)}\right)'=\left(\dfrac{x+2}{2-x}\right)'=\dfrac{2-x+x+2}{(2-x)^2}=\dfrac{4}{(2-x)^2}$;

**2)** $f'(x)=\left(\dfrac{(3x^2-1)^2-(x^2+1)^2}{4x^2-4x^4}\right)'=\left(\dfrac{9x^4-6x^2+1-x^4-2x^2-1}{4x^2(1-x^2)}\right)'=\left(\dfrac{8x^4-8x^2}{4x^2(1-x^2)}\right)'=\left(\dfrac{8x^2(x^2-1)}{4x^2(1-x)(1+x)}\right)'=\left(\dfrac{2(x-1)(x+1)}{-(x-1)(x+1)}\right)'=(-2)'=0$;

**3)** $f'(x)=\left(\dfrac{(18x^5-2x)^2}{9x^4+6x^2+1}\right)'=\left(\dfrac{4x^2(9x^4-1)^2}{(3x^2+1)^2}\right)'=\left(\dfrac{4x^2(3x^2-1)^2(3x^2+1)^2}{(3x^2+1)^2}\right)'=[4x^2(3x^2-1)^2]'=[(6x^3-2x)^2]'=2(6x^3-2x)\cdot(18x^2-2)=8x(3x^2-1)(9x^2-1)$;

**4)** $f'(x)=\left(\dfrac{1}{\sqrt[3]{x+\sqrt{x}}}\right)'=[(x+\sqrt{x})^{-\frac{1}{3}}]'=-\dfrac{1}{3}(x+\sqrt{x})^{-\frac{4}{3}}\cdot\left(1+\dfrac{1}{2\sqrt{x}}\right)=-\dfrac{\dfrac{2\sqrt{x}+1}{2\sqrt{x}}}{3(x+\sqrt{x})^{\frac{4}{3}}}=-\dfrac{2\sqrt{x}+1}{6\sqrt{x}(x+\sqrt{x})\sqrt[3]{x+\sqrt{x}}}$;

**5)** $f'(x)=(\sqrt[3]{1+x\sqrt{x+3}})'=[(1+x\sqrt{x+3})^{\frac{1}{3}}]'=\dfrac{1}{3}(1+x\sqrt{x+3})^{-\frac{2}{3}}\cdot\left(\sqrt{x+3}+\dfrac{x}{2\sqrt{x+3}}\right)=\dfrac{1}{3(1+x\sqrt{x+3})^{\frac{2}{3}}}\cdot\dfrac{2(x+3)+x}{2\sqrt{x+3}}=\dfrac{x+1}{2\sqrt{x+3}\sqrt[3]{(1+x\sqrt{x+3})^2}}$;

**6)** $f'(x)=\left(x\sqrt{\dfrac{1-x}{1+x^2}}\right)'=\sqrt{\dfrac{1-x}{1+x^2}}+x\cdot\dfrac{1}{2\sqrt{\dfrac{1-x}{1+x^2}}}\cdot\left(\dfrac{1-x}{1+x^2}\right)'=\sqrt{\dfrac{1-x}{1+x^2}}+\dfrac{x\sqrt{1+x^2}}{2\sqrt{1-x}}\cdot\dfrac{-1-x^2-2x+2x^2}{(1+x^2)^2}=\sqrt{\dfrac{1-x}{1+x^2}}+\dfrac{\sqrt{1+x^2}}{\sqrt{1-x}}\cdot\dfrac{x}{2}$.

$$\frac{x^2-2x-1}{\sqrt{1+x^2}\cdot\sqrt{1+x^2}\cdot(1+x^2)}=\sqrt{\frac{1-x}{1+x^2}}+\frac{x}{2}\cdot\frac{\sqrt{1-x}}{\sqrt{1+x^2}}\cdot\frac{x^2-2x-1}{(1-x)(1+x^2)}=$$

$$\sqrt{\frac{1-x}{1+x^2}}\left(1+\frac{x}{2}\cdot\frac{x^2-2x-1}{(1-x)(1+x^2)}\right)=\sqrt{\frac{1-x}{1+x^2}}\left(\frac{2(1+x^2-x-x^3)+x^3-2x-x}{2(x-1)(1+x^2)}\right)$$

$$\sqrt{\frac{1-x}{1+x^2}}\left(\frac{2+2x^2-2x-2x^3+x^3-2x^2-x}{2(1-x)(1+x^2)}\right)=\sqrt{\frac{1-x}{1+x^2}}\cdot\frac{2-x^3-3x}{2(1-x)(1+x^2)}.$$