

**Zadatak 2.** Odredi derivaciju inverzne funkcije  $f^{-1}(x)$  ako je:

$$1) f(x) = \frac{x+1}{2x-3};$$

$$2) f(x) = \log_2(x-1);$$

$$3) f(x) = \frac{1-e^x}{e^x};$$

$$4) f(x) = 1 - \ln \frac{x-1}{2}.$$

*Rješenje.*

$$1) x = \frac{y+1}{2y-3} \implies y+1 = 2xy-3x \implies y(1-2x) = -1-3x$$

$$\implies f^{-1}(x) = \frac{3x+1}{2x-1},$$

$$(f^{-1})'(x) = \frac{3(2x-1) - (3x+1) \cdot 2}{(2x-1)^2} = \frac{6x-3-6x-2}{(2x-1)^2} = -\frac{5}{(2x-1)^2};$$

$$2) f^{-1}(x) = 2^x + 1, \quad (f^{-1})'(x) = 2^x \ln 2;$$

$$3) x = e^{-y} - 1 \implies e^{-y} = x+1 \implies e^y = \frac{1}{x+1} \implies y = \ln \frac{1}{1+x} = -\ln|x+1|,$$

$$(f^{-1})'(x) = -\frac{1}{x+1};$$

$$4) x = 1 - \ln \frac{y-1}{2} \implies \ln \frac{y-1}{2} = 1-x = \frac{y-1}{2} = e^{1-x} \implies y-1 =$$

$$2e^{1-x} \implies f^{-1}(x) = 1 + 2e^{1-x},$$

$$(f^{-1})'(x) = 2e^{1-x}(-1) = -2e^{1-x}.$$