

**Zadatak 4.** Odredi derivacije sljedećih funkcija:

1)  $f(x) = x^2 \cdot \ln^2 x$ ;

2)  $f(x) = \frac{\ln^2 x}{x^2}$ ;

3)  $f(x) = \ln \frac{1 - \sqrt{x}}{1 + \sqrt{x}}$ ;

4)  $f(x) = \ln \left(1 + \frac{1}{x}\right)$ ;

5)  $f(x) = \ln(1 + \ln x)$ ;

6)  $f(x) = \frac{1}{2} \ln \sqrt{\frac{1+x}{1-x}}$ .

**Rješenje.**

1)  $f'(x) = 2x \cdot \ln^2 x + x^2 \cdot 2 \ln x \cdot \frac{1}{x} = 2x(\ln^2 x + \ln x)$ ;

2)  $f'(x) = \frac{2 \ln x \cdot \frac{1}{x} \cdot x^2 - 2x \ln^2 x}{x^4} = \frac{2x(\ln x - \ln^2 x)}{x^4} = \frac{2(\ln x - \ln^2 x)}{x^3}$ ;

3)  $f'(x) = \frac{1}{1 - \sqrt{x}} \cdot \left(\frac{1 - \sqrt{x}}{1 + \sqrt{x}}\right)' = \frac{1 + \sqrt{x}}{1 - \sqrt{x}} \cdot \frac{(1 - \sqrt{x})'(1 + \sqrt{x}) - (1 - \sqrt{x})(1 + \sqrt{x})'}{(1 + \sqrt{x})^2} =$

$$-\frac{1}{2\sqrt{x}}(1 + \sqrt{x}) - (1 - \sqrt{x}) \cdot \frac{1}{2\sqrt{x}} = \frac{-1 - \sqrt{x} - 1 + \sqrt{x}}{2\sqrt{x}} = -\frac{1}{\sqrt{x}(1 - x)};$$

4)  $f'(x) = \frac{1}{1 + \frac{1}{x}} \cdot \left(1 + \frac{1}{x}\right)' = \frac{1}{\frac{x+1}{x}} \cdot \frac{-1}{x^2} = -\frac{x}{x+1} \cdot \frac{1}{x^2} = -\frac{1}{x(x+1)}$ ;

5)  $f'(x) = \frac{1}{1 + \ln x} \cdot \frac{1}{x} = \frac{1}{x(1 + \ln x)}$ ;

6)  $f'(x) = \frac{1}{2\sqrt{\frac{1+x}{1-x}}} \cdot \frac{1}{2\sqrt{\frac{1+x}{1-x}}} \cdot \frac{1-x+1+x}{(1-x)^2} = \frac{1-x}{2(1+x)} \cdot \frac{1}{(1-x)^2} =$

$$\frac{1}{2(1+x)(1-x)} = \frac{1}{2(1-x^2)}.$$