Zadatak 7. Odredi prve četiri derivacije funkcija:

1) $f(x) = \dfrac{1}{x}$; **2)** $f(x) = \ln x$;

3) $f(x) = \ln(x+1)$; **4)** $f(x) = \ln\dfrac{1}{x}$;

5) $f(x) = x \ln x$; **6)** $f(x) = x^2 \ln x$.

Rješenje. **1)** $f'(x) = -\dfrac{1}{x^2}$, $\quad f''(x) = 2x^{-3} = \dfrac{2!}{x^3}$, $\quad f'''(x) = 2 \cdot (-3)x^{-4} = -\dfrac{3!}{x^4}$,

$f^{IV}(x) = 3 \cdot 4x^{-5} = \dfrac{4!}{x^5}$, $\quad f^{(n)}(x) = \dfrac{(-1)^n n!}{x^{n+1}}$;

2) $f'(x) = \dfrac{1}{x}$, $\quad f''(x) = -\dfrac{1}{x^2}$, $\quad f'''(x) = \dfrac{2!}{x^3}$,

$f^{IV}(x) = -\dfrac{3!}{x^4}$, $\quad f^n(x) = \dfrac{(-1)^{n+1}(n-1)}{x^n}$;

3) $f'(x) = \dfrac{2}{(x+1)^2}$, $\quad f''(x) = -\dfrac{2 \cdot 2}{(x+1)^3}$, $\quad f'''(x) = \dfrac{2 \cdot 3!}{(x+1)^4}$,

$f^{IV}(x) = -\dfrac{2 \cdot 4!}{(x+1)^5}$, $\quad f^n(x) = (-1)^{n+1}\dfrac{2 \cdot n!}{(x+1)^{n+1}}$;

4) $f'(x) = -\dfrac{1}{(1+x)^2}$, $\quad f''(x) = \dfrac{2!}{(1+x)^3}$, $\quad f'''(x) = -\dfrac{3!}{(1+x)^4}$,

$f^{IV}(x) = \dfrac{4!}{(1+x)^5}$, $\quad f^{(n)}(x) = \dfrac{(-1)^n n!}{(1+x)^{n+1}}$;

5) $f'(x) = 1 + \ln x$, $\quad f''(x) = \dfrac{1}{x}$, $\quad f'''(x) = -\dfrac{1}{x^2}$,

$f^{IV}(x) = \dfrac{2}{x^3}$, $\quad f^n(x) = \dfrac{(-1)^n(n-2)!}{x^{n-1}}$;

6) $f'(x) = \sin x + x \cos x$, $\quad f''(x) = 2\cos x - x \sin x$,

$f'''(x) = -3\sin x - x \cos x$, $\quad f^{IV}(x) = -4\cos x + x \sin x$.