

Zadatak 8. Deriviraj funkcije:

1) $f(x) = e^{2x+3}$;

2) $f(x) = e^{-x^2+2}$;

3) $f(x) = xe^{x^2}$;

4) $f(x) = \ln \frac{1}{x}$;

5) $f(x) = \ln(x\sqrt{x})$;

6) $f(x) = \ln \sqrt[n]{x}$;

7) $f(x) = \ln \sqrt{e^x}$;

8) $f(x) = \sqrt{\ln x}$;

9) $f(x) = \ln^2 x$;

10) $f(x) = \frac{1}{\ln x}$;

11) $f(x) = \ln(\sin x)$;

12) $f(x) = \ln(\cos x)$;

13) $f(x) = \frac{1}{2} \ln \frac{1-x}{1+x}$;

14) $f(x) = \ln(x + \sqrt{x^2 - 1})$.

Rješenje.

1) $f'(x) = (e^{2x+3})' = e^{2x+3} \cdot 2 = 2e^{2x+3}$;

2) $f'(x) = (e^{-x^2+2})' = e^{-x^2+2} \cdot (-2x) = -2xe^{-x^2+2}$;

3) $f'(x) = (xe^{x^2})' = e^{x^2} + x \cdot e^{x^2} \cdot 2x = (1 + 2x^2)e^{x^2}$;

4) $f'(x) = \left(\ln \frac{1}{x}\right)' = (-\ln x)' = -\frac{1}{x}$;

5) $f'(x) = [\ln(x\sqrt{x})]' = \left(\frac{3}{2} \ln x\right)' = \frac{3}{2x}$;

6) $f'(x) = (\ln \sqrt[n]{x})' = \left(\frac{1}{n} \ln x\right)' = \frac{1}{nx}$;

7) $f'(x) = (\ln \sqrt{e^x})' = \left(\frac{1}{2}x\right)' = \frac{1}{2}$;

8) $f'(x) = (\sqrt{\ln x})' = \frac{1}{2\sqrt{\ln x}} \cdot \frac{1}{x} = \frac{1}{2x\sqrt{\ln x}}$;

9) $f'(x) = (\ln^2 x)' = 2 \ln x \cdot \frac{1}{x} = \frac{2 \ln x}{x}$;

10) $f'(x) = \left(\frac{1}{\ln x}\right)' = -\frac{1}{\ln^2 x} \cdot \frac{1}{x} = -\frac{1}{x \ln^2 x}$;

11) $f'(x) = [\ln(\sin x)]' = \frac{1}{\sin x} \cos x = \operatorname{ctg} x$;

12) $f'(x) = [\ln(\cos x)]' = \frac{1}{\cos x} (-\sin x) = -\operatorname{tg} x$;

13) $f'(x) = \left(\frac{1}{2} \ln \frac{1-x}{1+x}\right)' = \left[\frac{1}{2} \ln(1-x) - \frac{1}{2} \ln(1+x)\right]' = -\frac{1}{2(1-x)} -$

$\frac{1}{2(1+x)} = \frac{-1-x-1+x}{2(1-x^2)} = \frac{1}{x^2-1}$;

14) $f'(x) = [\ln(x + \sqrt{x^2 - 1})]' = \frac{1}{x + \sqrt{x^2 - 1}} \cdot \left(1 + \frac{2x}{2\sqrt{x^2 - 1}}\right) =$

$\frac{1}{x + \sqrt{x^2 - 1}} \cdot \frac{\sqrt{x^2 - 1} + x}{\sqrt{x^2 - 1}} = \frac{1}{\sqrt{x^2 - 1}}$.