

Zadatak 9. Deriviraj funkcije:

$$1) f(x) = \frac{1}{\ln^2 x};$$

$$2) f(x) = \ln \frac{1+x^2}{1-x^2};$$

$$3) f(x) = \frac{1}{2} \ln \sqrt{\frac{1+x}{1-x}};$$

$$4) f(x) = \ln \frac{x + \sqrt{x^2 + 1}}{\sqrt{x^2 + 1} - x};$$

$$5) f(x) = \ln \ln(x^4 + x);$$

$$6) f(x) = \ln(\ln x);$$

$$7) f(x) = \ln(\sqrt{1+x^2} - x) + \ln(\sqrt{1+x^2} + x).$$

Rješenje.

$$1) f'(x) = \left(\frac{1}{\ln^2 x} \right)' = -\frac{2}{\ln^3 x} \cdot \frac{1}{x} = -\frac{2}{x \ln^3 x};$$

$$2) f'(x) = \left(\ln \frac{1+x^2}{1-x^2} \right)' = \frac{1-x^2}{1+x^2} \cdot \frac{2x(1-x^2) - (1+x^2)(-2x)}{(1-x^2)^2} = \frac{2x(1-x^2+1+x^2)}{1-x^4} = \frac{4x}{1-x^4};$$

$$3) f'(x) = \left(\frac{1}{2} \ln \sqrt{\frac{1+x}{1-x}} \right)' = \left(\frac{1}{4} \ln \frac{1+x}{1-x} \right)' = \left[\frac{1}{4} \ln(1+x) - \frac{1}{4} \ln(1-x) \right]' = \frac{1}{4(1+x)} + \frac{1}{4(1-x)} = \frac{2}{4(1-x^2)} = \frac{1}{2(1-x^2)};$$

$$4) f'(x) = \left(\ln \frac{x + \sqrt{x^2 + 1}}{\sqrt{x^2 + 1} - x} \right)' = \left[\ln(x + \sqrt{x^2 + 1}) - \ln(\sqrt{x^2 + 1} - x) \right]' = \frac{1}{x + \sqrt{x^2 + 1}} \cdot \left(1 + \frac{x}{\sqrt{x^2 + 1}} \right) - \frac{1}{\sqrt{x^2 + 1} - x} \cdot \left(\frac{x}{\sqrt{x^2 + 1}} \right) = \frac{1}{\sqrt{x^2 + 1}} + \frac{1}{\sqrt{x^2 + 1}} = \frac{2}{\sqrt{x^2 + 1}};$$

$$5) f'(x) = [\ln \ln(x^4 + x)]' = \frac{1}{\ln(x^4 + x)} \cdot \frac{1}{x^4 + x} \cdot (4x^3 + 1) = \frac{4x^3 + 1}{(x^4 + x) \ln(x^4 + x)};$$

$$6) f'(x) = [\ln(\ln x)]' = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x};$$

$$7) f'(x) = [\ln(\sqrt{1+x^2} - x) + \ln(\sqrt{1+x^2} + x)]' = [\ln(1+x^2 - x^2)]' = (\ln 1)' = 0' = 0.$$