Zadatak 11.   Deriviraj sljedeće funkcije:

1) $f(x) = \ln \dfrac{1}{\sin^2 x}$;

2) $f(x) = \ln \cos \dfrac{x-1}{x}$;

3) $f(x) = \ln \operatorname{tg} \dfrac{x}{2}$;

4) $f(x) = \ln \operatorname{tg}\left(\dfrac{x}{2} + \dfrac{\pi}{4}\right)$;

5) $f(x) = \ln e^{1-\cos^2 x}$;

6) $f(x) = \log_{\frac{1}{e}}(\sin x)$;

7) $f(x) = \log_{e^2}(\cos x)$;

8) $f(x) = \log_{\sqrt{e}} \dfrac{1}{\cos^2 x}$.

Rješenje.   **1)** $f'(x) = \left(\ln \dfrac{1}{\sin^2 x}\right)' = [-2\ln(\sin x)]' = -\dfrac{2}{\sin x}\cdot \cos x = -2\operatorname{ctg} x$;

2) $f'(x) = \left(\ln\cos\dfrac{x-1}{x}\right)' = \dfrac{1}{\cos\dfrac{x-1}{x}}\cdot\left(-\sin\dfrac{x-1}{x}\right)\cdot\left(\dfrac{1}{x^2}\right) = -\dfrac{1}{x^2}\operatorname{tg}\dfrac{x-1}{x}$;

3) $f'(x) = \left(\ln\operatorname{tg}\dfrac{x}{2}\right)' = \dfrac{1}{\operatorname{tg}\dfrac{x}{2}}\cdot\dfrac{1}{\cos^2\dfrac{x}{2}}\cdot\dfrac{1}{2} = \dfrac{\cos\dfrac{x}{2}}{\sin\dfrac{x}{2}}\cdot\dfrac{1}{\cos^2\dfrac{x}{2}}\cdot\dfrac{1}{2} = \dfrac{1}{\sin x}$;

4) $f'(x) = \left[\ln\operatorname{tg}\left(\dfrac{x}{2}+\dfrac{\pi}{4}\right)\right]' = \dfrac{1}{\operatorname{tg}\left(\dfrac{x}{2}+\dfrac{\pi}{4}\right)}\cdot\dfrac{1}{\cos^2\left(\dfrac{x}{2}+\dfrac{\pi}{4}\right)}\cdot\dfrac{1}{2} = \dfrac{1}{\sin\left(x+\dfrac{\pi}{2}\right)} =$

$\dfrac{1}{\cos x}$;

5) $f'(x) = (\ln e^{1-\cos^2 x})' = (1-\cos^2 x)' = (\sin^2 x)' = 2\sin x\cos x = \sin 2x$;

6) $f'(x) = [\log_{\frac{1}{e}}(\sin x)]' = [-\ln(\sin x)]' = -\dfrac{1}{\sin x}\cos x = -\operatorname{ctg} x$;

7) $f'(x) = [\log_{e^2}(\cos x)]' = \left[\dfrac{1}{2}\ln(\cos x)\right]' = \dfrac{1}{2}\cdot\dfrac{1}{\cos x}\cdot(-\sin x) = -\dfrac{1}{2}\operatorname{tg} x$;

8) $f'(x) = \left(\log_{\sqrt{e}}\dfrac{1}{\cos^2 x}\right)' = [2\ln(\cos^{-2} x)]' = [-4\ln(\cos x)]' = -\dfrac{4}{\cos x}\cdot$

$(-\sin x) = 4\operatorname{tg} x$.