

Zadatak 12. Deriviraj funkcije:

$$1) f(x) = e^{\ln \sin 5x};$$

$$2) f(x) = \left(\frac{1}{3}\right)^{-\log_9(x-1)};$$

$$3) f(x) = 4^{-\log_2 \sqrt{1-x^2}};$$

$$4) f(x) = \left(\frac{1}{\sqrt{e}}\right)^{-\ln(x^2+x)};$$

$$5) f(x) = \log_2 \sqrt{\frac{1-\sin 2x}{1+\sin 2x}};$$

$$6) f(x) = \log_x(x+2).$$

$$\text{Rješenje. } 1) f'(x) = (e^{\ln \sin 5x})' = (\sin 5x)' = 5 \cos 5x;$$

$$2) f'(x) = \left[\left(\frac{1}{3} \right)^{-\log_9(x-1)} \right]' = [3^{\frac{1}{2} \log_3(x-1)}]' = (\sqrt{x-1})' = \frac{1}{2\sqrt{x-1}};$$

$$3) f'(x) = (4^{-\log_2 \sqrt{1-x^2}})' = [2^{-\log_2(1-x^2)}]' = \left(\frac{1}{1-x^2} \right)' = [(1-x^2)^{-1}]' = -\frac{1}{(1-x^2)^2} \cdot (-2x) = \frac{2x}{(1-x^2)^2};$$

$$4) f'(x) = \left[\left(\frac{1}{\sqrt{e}} \right)^{-\ln(x^2+x)} \right]' = [e^{\frac{1}{2} \ln(x^2+x)}]' = (\sqrt{x^2+x})' = \frac{1}{2\sqrt{x^2+x}} \cdot (2x+1) = \frac{2x+1}{2\sqrt{x^2+x}};$$

$$5) f'(x) = \left(\log_2 \sqrt{\frac{1-\sin 2x}{1+\sin 2x}} \right)' = \left(\frac{1}{2} \log_2 \frac{1-\sin 2x}{1+\sin 2x} \right)' = \left[\frac{1}{2} \log_2 \frac{(\sin x - \cos x)^2}{(\sin x + \cos x)^2} \right]' = \left(\log_2 \frac{\sin x - \cos x}{\sin x + \cos x} \right)' = [\log_2(\sin x - \cos x) - \log_2(\sin x + \cos x)]' = \frac{1}{\ln 2} \left[\frac{\cos x + \sin x}{\sin x - \cos x} - \frac{\cos x - \sin x}{\sin x + \cos x} \right] = \frac{1}{\ln 2} \left[\frac{\sin x + \cos x}{\sin x - \cos x} + \frac{\sin x - \cos x}{\sin x + \cos x} \right] = \frac{\frac{1}{\ln 2} \frac{\sin^2 x + \cos^2 x + \sin 2x + \sin^2 x + \cos^2 x - \sin 2x}{\sin^2 x - \cos^2 x}}{\ln 2} = -\frac{2}{\ln 2 \cos 2x};$$

$$6) f'(x) = [\log_x(x+2)]' = \left[\frac{\ln(x+2)}{\ln x} \right]' = \frac{\frac{1}{x+2} \cdot \ln x - \frac{1}{x} \ln(x+2)}{\ln^2 x} = \frac{x \ln x - (x+2) \ln(x+2)}{x(x+2) \ln^2 x};$$

$$7) f'(x) = \left(\log_{1-x} \frac{2x+3}{x^2} \right)' = [\log_{1-x}(2x+3) - \log_{1-x} x^2]' = \left[\frac{\ln(2x+3)}{\ln(1-x)} - \frac{2 \ln x}{\ln(1-x)} \right]'$$

$$= \frac{\frac{2}{2x+3} \ln(1-x) + \frac{1}{1-x} \ln(2x+3)}{\ln^2(1-x)} - \frac{\frac{2}{x} \ln(1-x) + \frac{2}{1-x} \ln x}{\ln^2(1-x)} = \frac{1}{\ln^2(1-x)}.$$

$$\left[\left(\frac{2}{2x+3} - \frac{2}{x} \right) \ln(1-x) + \frac{1}{1-x} \ln(2x+3) - \frac{2}{1-x} \ln x \right] = \frac{1}{\ln^2(1-x)} \left[\frac{1}{1-x} \right].$$

$$\ln \left(\frac{2x+3}{x^2} \right) - \frac{2x+6}{x(2x+3)} \ln(1-x);$$

$$\begin{aligned} \mathbf{8)} \quad f'(x) &= (\log_2 \log_3 x)' = \left(\log_2 \frac{\ln x}{\ln 3} \right)' = (\log_2 \ln x - \log_2 \ln 3)' \\ &= \left(\frac{\ln \ln x}{\ln 2} - \log_2 \ln 3 \right)' = \frac{1}{\ln 2} \cdot \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln 2 \ln x}. \end{aligned}$$