

Zadatak 13. Deriviraj funkcije:

1) $f(x) = \log(x+1)$;

2) $f(x) = \log_{\frac{1}{2}} \frac{1}{x}$;

3) $f(x) = \log_2(1 + \log_3 x)$;

4) $f(x) = x \log x$;

5) $f(x) = \log_2 \log_3 x$;

6) $f(x) = \log\left(1 + \frac{1}{x}\right)$.

Rješenje.

1) $f'(x) = [\log(x+1)]' = \frac{1}{(x+1) \ln 10}$;

2) $f'(x) = \left(\log_{\frac{1}{2}} \frac{1}{x}\right)' = (\log_2 x)' = \frac{1}{x \ln 2}$;

3) $f'(x) = [\log_2(1 + \log_3 x)]' = \frac{1}{(1 + \log_3 x) \ln 2} \cdot \frac{1}{x \ln 3} = \frac{1}{\left(1 + \frac{\ln x}{\ln 3}\right) \ln 2}$.

$\frac{1}{x \ln 3} = \frac{1}{\frac{\ln 3x \ln 2}{\ln 3}} \cdot \frac{1}{x \ln 3} = \frac{1}{x \ln 3x \ln 2}$;

4) $f'(x) = (x \log x)' = \log x + \frac{1}{\ln 10} = \frac{\ln x}{\ln 10} + \frac{1}{\ln 10} = \frac{1 + \ln x}{\ln 10}$;

5) $f'(x) = (\log_2 \ln x)' = \frac{1}{\ln x} \cdot \frac{1}{\ln 2} \cdot \frac{1}{x} = \frac{1}{x \ln 2 \ln x}$;

6) $f'(x) = \left[\log\left(1 + \frac{1}{x}\right)\right]' = [\log(x+1) - \log x]' = \frac{1}{(x+1) \ln 10} - \frac{1}{x \ln 10} = \frac{x - x - 1}{x(x+1) \ln 10} = -\frac{1}{x(x+1) \ln 10}$.