Zadatak 15. Deriviraj sljedeće funkcije:

1) $f(x) = \ln \frac{\sqrt{4\operatorname{tg} x + 1} - 2\sqrt{\operatorname{tg} x}}{\sqrt{4\operatorname{tg} x + 1} + 2\sqrt{\operatorname{tg} x}}$;

2) $f(x) = \ln \frac{x \ln x - 1}{x \ln x + 1}$;

3) $f(x) = \ln \frac{\sqrt{x^4+1} - x^2}{\sqrt{x^4+1} + x^2}$.

Rješenje. **1)**

$$f'(x) = \left(\ln \frac{\sqrt{4\operatorname{tg} x + 1} - 2\sqrt{\operatorname{tg} x}}{\sqrt{4\operatorname{tg} x + 1} + 2\sqrt{\operatorname{tg} x}}\right)'$$

$$= \left[\ln\left(\frac{\sqrt{4\operatorname{tg} x + 1} - 2\sqrt{\operatorname{tg} x}}{\sqrt{4\operatorname{tg} x + 1} + 2\sqrt{\operatorname{tg} x}} \cdot \frac{\sqrt{4\operatorname{tg} x + 1} - 2\sqrt{\operatorname{tg} x}}{\sqrt{4\operatorname{tg} x + 1} - 2\sqrt{\operatorname{tg} x}}\right)\right]' = \left[\ln \frac{(\sqrt{4\operatorname{tg} x + 1} - 2\sqrt{\operatorname{tg} x})^2}{4\operatorname{tg} x + 1 - 4\operatorname{tg} x}\right]'$$

$$= [\ln(\sqrt{4\operatorname{tg} x + 1} - 2\sqrt{\operatorname{tg} x})^2]' = 2[\ln(\sqrt{4\operatorname{tg} x + 1} - 2\sqrt{\operatorname{tg} x})]'$$

$$= \frac{2}{\sqrt{4\operatorname{tg} x + 1} - 2\sqrt{\operatorname{tg} x}} \cdot (\sqrt{4\operatorname{tg} x + 1} - 2\sqrt{\operatorname{tg} x})'$$

$$= \frac{2}{\sqrt{4\operatorname{tg} x + 1} - 2\sqrt{\operatorname{tg} x}} \cdot \left(\frac{1}{2\sqrt{4\operatorname{tg} x + 1}} \cdot (4\operatorname{tg} x + 1)' - \frac{1}{\sqrt{\operatorname{tg} x}} \cdot (\operatorname{tg} x)'\right)$$

$$= \frac{2}{\sqrt{4\operatorname{tg} x + 1} - 2\sqrt{\operatorname{tg} x}} \cdot \left(\frac{2}{\sqrt{4\operatorname{tg} x + 1}} \cdot \frac{1}{\cos^2 x} - \frac{1}{\sqrt{\operatorname{tg} x}} \cdot \frac{1}{\cos^2 x}\right)$$

$$= \frac{2}{\cos^2 x}\left[\frac{1}{\sqrt{4\operatorname{tg} x + 1} - 2\sqrt{\operatorname{tg} x}} \cdot \frac{2\sqrt{\operatorname{tg} x} - \sqrt{4\operatorname{tg} + 1}}{\sqrt{4\operatorname{tg} x + 1} - 2\sqrt{\operatorname{tg} x}}\right]$$

$$= -\frac{2}{\cos^2 x\sqrt{\operatorname{tg} x(4\operatorname{tg} x + 1)}};$$

2) $f'(x) = \ln \frac{x \ln x - 1}{x \ln x + 1} = \frac{2}{x\sqrt{1+x^2}}$;

3)

$$f'(x) = \left(\ln \frac{\sqrt{x^4+1} - x^2}{\sqrt{x^4+1} + x^2}\right)' = \left[\ln\left(\frac{\sqrt{x^4+1} - x^2}{\sqrt{x^4+1} + x^2} \cdot \frac{\sqrt{x^4+1} - x^2}{\sqrt{x^4+1} - x^2}\right)\right]' =$$

$$\left[\ln \frac{(\sqrt{x^4+1} - x^2)^2}{x^4 + 1 - x^4}\right]' = [\ln(\sqrt{x^4+1} - x^2)^2]' = 2[\ln(\sqrt{x^4+1} - x^2)]' = 2 \cdot$$

$$\frac{1}{\sqrt{x^4+1} - x^2} \cdot (\sqrt{x^4+1} - x^2)' = \frac{2}{\sqrt{x^4+1} - x^2} \cdot \left(\frac{1}{2\sqrt{x^4+1}} \cdot 4x^3 - 2x\right) =$$

$$\frac{2}{\sqrt{x^4+1} - x^2} \cdot \frac{2x^3 - 2x\sqrt{x^4+1}}{\sqrt{x^4+1}} = \frac{4x}{\sqrt{x^4+1} - x^2} \cdot \frac{x^2 - \sqrt{x^4+1}}{\sqrt{x^4+1}} = -\frac{4x}{\sqrt{x^4+1}}.$$